Mathlete Training Centre

RMO 2025 Round 1

1. Calculate $\dfrac{1008}{24} + \dfrac{1008}{40} + \dfrac{1008}{60} + \dfrac{1008}{84} + \dfrac{1008}{112} + \dfrac{1008}{144} + \dfrac{1008}{180} + \dfrac{1008}{220} + \dfrac{1008}{264}$

2. There are 25 students whose sum of scores is 2025 and no two students have the same score. If the highest score among them is 94, what is the value of the lowest possible score?

3. A, B, C and D know how many marbles each of them has and each says a statement as shown below.

   A: I have neither the most nor the least.

   B: I have not the least.

   C: I have the most.

   D: I have the least.

   Given that only one of them is lying, who is lying?

4. $5 \times 7^3 \times 35$ can be written as x^y. What is the minimum value of x+y?

5. What is the number of trailing zeros for $1 \times 2 \times 3 \times 4 \times 5 \times ... \times 2025$?

6. What is the value of $\dfrac{1025 \times 2025}{410 + 205^2} + \dfrac{1025 \times 2025}{414 + 207^2} + ... + \dfrac{1025 \times 2025}{450 + 225^2}$
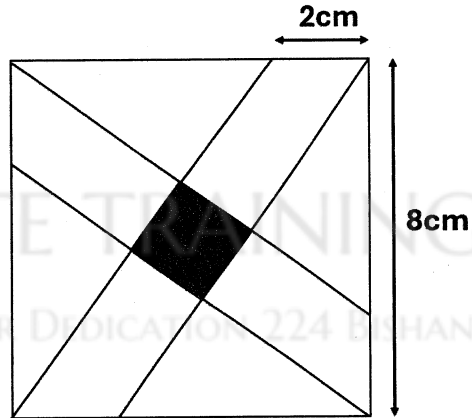
7. The side length of the square is 8cm, find the fraction of the shaded area to the square.

2cm

8cm

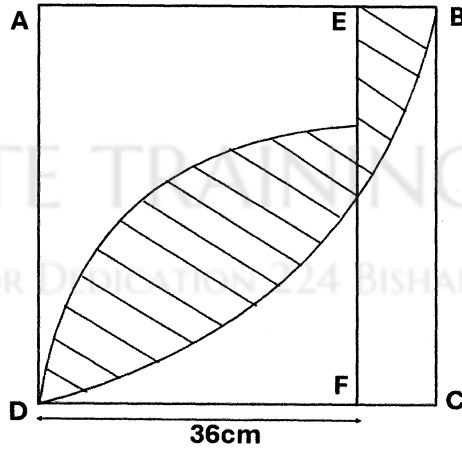8. Given $\angle ACD = 35°$ and $\angle AGB = 6°$. ADG and CDE are straight lines and are perpendicular to each other, and lines BE and CF are equal in length. Find $\angle CEF$.

A   B

35°   D   E

6°

C

F   G
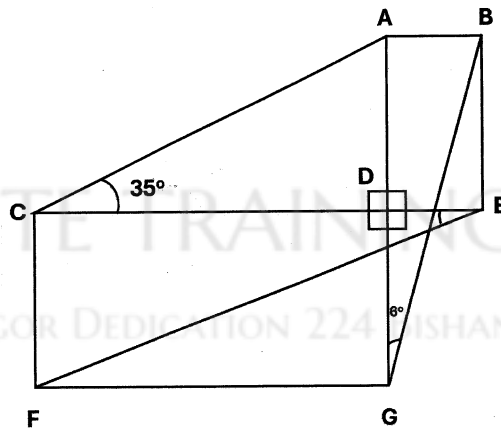
9. ABCD is a square and its area is 2025, DF is 36cm, find the area of the shaded region. (Take π as 3.)

10. There are a total of 100 red, blue, green and yellow marbles. The ratio of blue to green marbles is 5:1. The amount of green marbles evenly divides the amount of red marbles. After 134 blue marbles are added, the ratio of blue to red marbles is 9:1. How many yellow marbles are there?

11. $(1 + \frac{1}{3} + \frac{3}{5} + \frac{5}{7} + ... + \frac{2021}{2023}) \times (\frac{1}{3} + \frac{3}{5} + \frac{5}{7} + ... + \frac{2023}{2025}) - (1 + \frac{1}{3} + \frac{3}{5} + \frac{5}{7} + ... + \frac{2023}{2025})$
$\times (\frac{1}{3} + \frac{3}{5} + \frac{5}{7} + ... + \frac{2021}{2023})$

12. An 8 digit phone number $\overline{85a32b63}$ is divisible by 99. What is the value of 11a + 9b?

13. A 3 digit number is such that the digit in the tens place is the average of all the digits. How many such numbers are there?

14. $\dfrac{(3^2 + 5^2 + 7^2 + ... + 2025^2) - (2^2 + 4^2 + 6^2 + ... + 2024^2)}{(3 + 5 + 7 + ... + 2025) - (2 + 4 + 6 + ... + 2024)}$

15. A number is formed strarting with one '1's, two '2's, three '3's, and so on. What is the 2025th digit?

16. Someone has 42 marbles and gives away a prime number x amount of marbles and is left with a prime number amount of marbles. He is given a prime number y amount of marbles and is left with another prime number amount of marbles. He then gives away a prime number z amount of marbles and is left with a non-zero amount of marbles. How many combinations of (x, y, z) are there? (x, y, z are not necessarily distinct.)

17. There are 2 towns, X and Y. They both start travelling from X to y at 8am and Alice takes 2h 24min while Bob takes 3h. Alice turns back walking at the same speed after reaching Y, at what time will Alice and Bob meet?

18. There are 2 jobs, X and Y. For job X, C takes 10 days and D takes 7 days to complete it. For job Y, C takes 16 days to complete it and D takes 20 days to complete it. If they can work together or separetely, what is the minimum number of days required to complete both jobs?

19. There is a 9-digit number that includes all digits 1 to 9. It also has the following properties:
    (a) All square numbers digits are adjacent.
    (b) The sum of the first 6 digits is less than the sum of the last 3 digits.
    (c) The first 4 digits are divisible by 16 if it is an individual number.
    (d) All 2 consecutive digits cannot be both even or both odd.
    If the number meets all the conditions, what is the remainder when it is divided by 7?

20. There are a total of 60 blue and red balls in a bag, and when put into 6 boxes of 10 balls each, one box will have at least 4 black balls, and when put into 3 boxes of 20 balls each, each box will have at least 1 red ball. Find the number of red balls.