Mathlete Training Centre

2023 HXC S1

Section A: Question 1 to 10 (3 marks each, total 30 marks)

1. How many solution(s) does the equation $20(23x - 5) + 21 = 46(10x + 3) - 217$ about x have? (Answer with the corresponding letter.)

   (A) 1 $B)0$ C) infinitely many

2. Find the remainder when 521^{2023} is divided by 9.

3. There is a task. Hua, Xia and Bei can finish the task alone in 5x days, 2x days and x days respectively. If Hua, Xia and Bei do the task together, it takes 30 days. How many days does it take for Xia to finish the task alone?

4. For a natural number n which is not smaller than 3, $\overline{n}$ represent the smallest prime number which is relatively prime with n. Find the value of $\overline{2023} \times \overline{8} \times \overline{12} \times \overline{30}$.

5. Find the value of $1 \times 4 + 2 \times 5 + 3 \times 6 + ... + 20 \times 23$.

6. Refer to the figure, $CO \perp AB$, $\angle AOH = \angle COM = 20°$, $\angle HOX = \angle MOF = 30°$. How many pairs of complementary angles are there in total as shown in the figure?
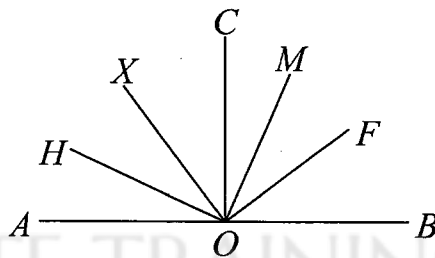
7. Four points A, B, C and D are collinear. AB = 23. AC = 5. BD = 21. Find the sum of the possible values of length of CD.
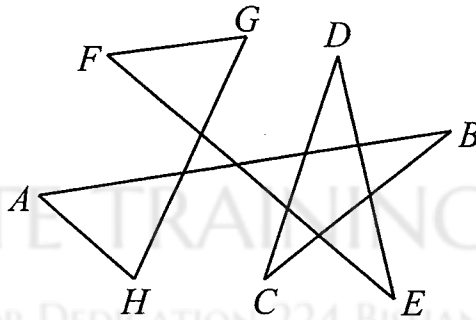
8. Let x and y be variables, a and b be unknown constant. Monomials $-\frac{20}{23}x^{2a+3}y^b$ and $5.21y^{2-a}x^{10-b}$ are liked terms. Find the value of 2a - 3b.

9. Refer to the figure, how many degrees is the sum of ∠A + ∠B + ∠C + ∠D + ∠E + ∠F + ∠G + ∠H ?

10. Refer to the figure, it is formed of 10 identical small rectangles. The perimeter of the whole figure is 220. Find the area of the whole figure.
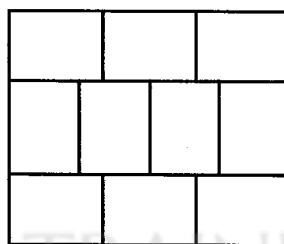
Section B: Question 11 to 20 (5 marks each, total 50 marks)

11. Find the sum of possible value(s) of $\dfrac{abc}{|abc|} - \dfrac{ab}{|ab|} + \dfrac{ac}{|ac|} + \dfrac{bc}{|bc|} - \dfrac{a}{|a|} - \dfrac{b}{|b|} + \dfrac{c}{|c|}$.

12. Let x, y and z be integers. The solution of inequality $5x+2 \le 1$ for x is $x \le a$. The solution of inequality $20y - 23 \ge 7$ for y is $y \ge b$. If $az - ab < bz$, find the minimum possible value of z.

13. Hua and Xia are playing a number-guessing game. First, Hua decides a number from 1 to 2023. Then Xia guesses the number. If he guesses it wrong, Hua must reveal the answer and decide a new number from the remaining numbers. Find the difference between the probabilities of Xia guessed correctly at the 5th trial and the 21st trial.

14. Hua has some 3-dollar, 5-dollar and 8-dollar stamps. There are 100 stamps in total. The number of 3-dollar stamps is not less than a half of the total. The amount of 5-dollar stamps is equal to the amount of 8-dollar stamps. At most how many dollars is the total amount of Hua's stamps.

15. Let a and b be positive integers. $20a + 25b = 325$. Find the sum of all possible values of a + b.

16. Refer to the figure, in a rectangle with an area of 1512 square centimetres, there is a rectangle with an area of 216 square centimetres and a square with an area of x square centimetres. Given a, b and c are all positive integers, find the value of x.

17. The ratio of the sizes of exterior angles of a convex quadrilateral is 3:4:5:6. How many degrees is the largest interior angle of the quadrilateral?
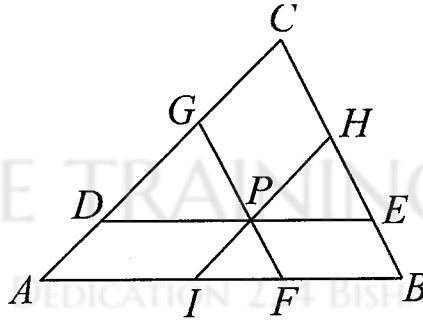
18. Find the value of $\dfrac{1}{20 \times 63} + \dfrac{1}{63 \times 43} + \dfrac{1}{43 \times 66} + \dfrac{1}{66 \times 23}$.

19. Refer to the figure, in △ABC, DE//AB. FG//BC. HI//CA. DE, FG and HI concurrent at P. The areas of quadrilateral GPHC, DPIA and PEBF are 60, 45 and 36 respectively. Find the area of △ABC.

20. Let x, a and b be real numbers. The solutions of equation $6(x + a) = 5x$ and $2(2x - a) = 3x - 2$ about x are both $x = b$. Find the value of b.

Section C: Question 21 to 25 (8 marks each, total 40 marks)

21. Find the remainder when $(1^2 + 3^2 + 5^2 + ... + 2023^2) + (1 + 2 + 3 + ... + 2024)$ is divided by 505.

22. 9 football teams join a single-round-robin tournament, i.e. every 2 teams compete exactly once. Every win gets 3 points, every draw gets 1 point, every loss gets 0 point. Finally, all 9 teams have different points. Team A has 2 wins and 1 loss. The highest and lowest possible rank of team A are X and Y respectively. Find the two-digit number $\overline{XY}$.

23. $\otimes$ is a calculation procedure defined as $a \otimes b = \dfrac{a^2 + 2023|b - 521|}{a}$, where a and b are rational numbers. If $2023 \otimes n = 10000$, find the sum of all possible value(s) of n.

24. The length of one side of a right-angled triangle is 23. The lengths of other two sides are also integers. Find the perimeter of the triangle.

25. Given that 1053983 has 12 positive factors. Find the sum of prime factors of 1053983. (The sum of prime factors of $60 = 2 + 3 + 5 = 10$.)

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