Mathlete Training Centre

SMOPS 2021 Round 1

1. Calculate $\dfrac{5 \times 6 \times 7 \times 8 \times 9 - 1 \times 2 \times 3 \times 4 \times 5}{1 \times 2 \times 3 \times 4 \times 5}$

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2. Thiago scored an average of 89 marks for his Chinese and Math test. His average score for his English and Math is 91. His average score for his English and Chinese is 84. What is his overall average score for all three subjects?

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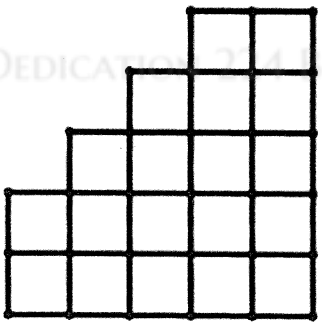
3. It is given that A, B, C are positive integers and $\frac{24}{5}=A+\cfrac{1}{B+\cfrac{1}{C+1}}$. Find the value of A+B+C.

4. In the table below, the first row and the first column are filled with integers from 0 to 9 as shown. For each of the remaining cells, the number in the cell is equal to the sum of the leftmost entry in its row and the topmost entry in its column. For example, x=4+3=7. Find the total sum of the 30 numbers in the table.

0	1	3	5	7	9
2					
4		x			
6					
8					

5. A four-digit number is divisible by 9. If its unit digit is removed, the remaining three-digit number is divisible by 4. Find the largest four-digit number that satisfy the condition.

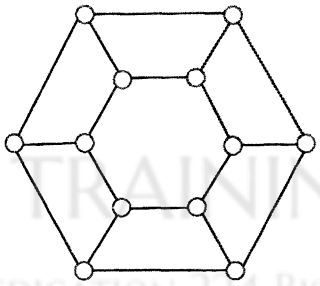
6. In the diagram below, how many ways are there to place 5 stones in 5 small squares such that there is exactly one stone in every row and every column?



7. Given that the sum of the four distinct positive integers is 1111, what is the largest possible value of their highest common factor?

8. On the whiteboard, David wrote n natural numbers, 1, 2, 3, 4, 5, ..., n, in a row to form a 3289-digit natural number 1234567891011... . What is the value of n?

9. In the following figure, all twelve circles are coloured such that, for each circle, its colour is the same as one of the three circles connected to it. Futhermore, the three circles connected to it have different colours. What is the smallest number of distinct colours needed to achieve this?



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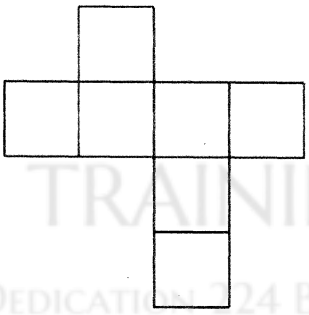
10. The figure below consists of 7 identical squares of side length 1. Given that the area of the smallest circle that completely contains the figure is $a\pi$, find the value of a.



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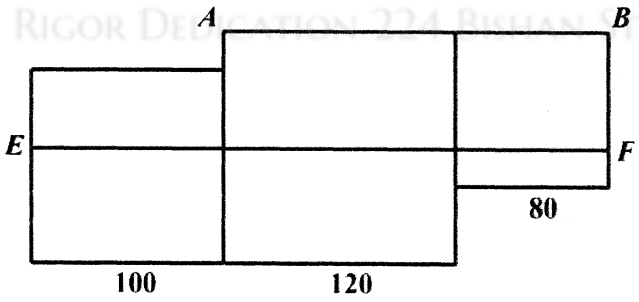
11. Observe the fractions $\frac{1}{8}, \frac{2}{9}, \frac{3}{10}, \frac{4}{11}, \frac{5}{12}, \ldots, \frac{2021}{2028}$. How many fractions are in their lowest term?

    (If a fraction is in its lowest term, the highest common factor of its numerator and denomiator is 1.)

12. Lisa solved 25 Math Olympiad problems over 6 days, with at least one problem solved on each day. It is also known that on each day she solved more problems than the previous day. How many different combinations are there for the daily number of questions solved over the 6 days?

13. Given that $1086624=2^5\times3^2\times7^3\times11^1$, how many factors of 108664 are divisible by 2 or 7 but not both?

14. Three squares of sides 100cm, 120cm and 80cm are placed next to one another as shown in the figure below. A segment EF, which is parallel to AB, divides the figure into 2 regions of equal area. Find the length of BF in cm.

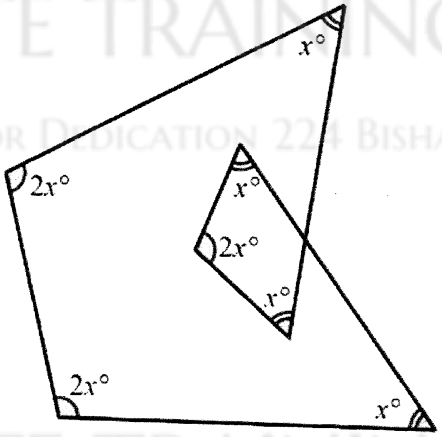


15. A 3-digit natural number X *overpowers* another 3-digit number Y if each of the digits of X is not less than the corresponding digit in Y. For example, 472 overpowers 370, but 472 does not overpower 228. How many 3-digit numbers overpower the number 587?

16. Four contestants took part in a chess tournament. Each person plays against every other player once. For each game, 2 point is awarded for a win, 0 for a loss and 1 point to each player for a draw. At the end of the tournament, no player has a perfect winning record, and each player has a different score. At most how many games ended in a draw?

17. Find the smallest positive integer n such that neither $4n$-1 not $4n$+1 is prime.

18. In the figure below, all angles are marked as shown. Find the value of x.

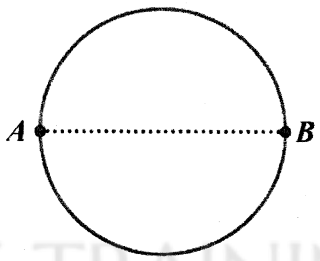


19. There are three green cards with numbers 1, 2, and 3 written on them respectively. Similarly, there are three red cards with numbers 1, 2 and 3 written on them respectively. How many ways are there to arrange the six cards in a row, such that the adjacent cards have different colours and different numbers?

20. Given that n is the smallest positive integer such that $7n$ has 2021 digits. Find the unit digit of n.

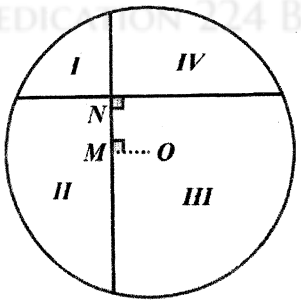
21. In year 2020, there were 1050 boys and 750 girls taking part in Singapore Mathematical Olympiad for Primary Schools (SMOPS), while 850 boys and 950 girls took part in Hwa Chong Infocomm Challenge (HCIC). As some students participated in both competitions, we know that there is a total of 2800 students who took part in at least one competition. Furthermore, the number of boys who took part in SMOPS only is equal to the number of girls who took part in HCIC only. How many girls took part in SMOPS only?

22. On a circular track, Lewis and Sebastian start walking from points A and B respectively at the same time. AB is the diameter of the circle. Lewis goes clockwise while Sebastian goes anticlockwise. It is known that when they meet in for the first time, Sebastian walked 100 metres. Later, when they meet for the second time, Lewis is 60 metres away from completing his first lap. Find the circumference of the track in metres.



23. A number is called *misplaced* is digit 1 is not the first digit counting from the left, digit 2 is not the secnod digit counting from the left, and so on. For example, 2341 is *misplaced.* 2134 is not misplaced because digit 3 is at the third place from the left. Find the number of four-digit *misplaced* numbers formed using digits 2, 3, 4 and 5 without repetition.

24. In the diagram below, a circle centred at O has radius 4cm. It is divided into 4 regions by two chords that are perpendicular to each other at point N. It is known that OM=1cm, MN=2cm. Find the value of Area of (I+III)-Area of (II+IV).

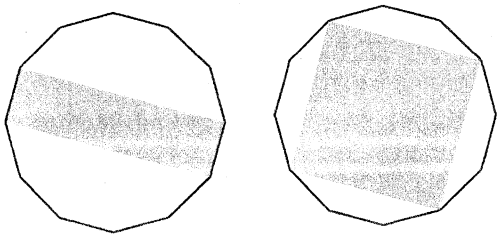


25. A palindrome is a number which reads the same backward as forward. For example, 232, 4774, 6 are palindromes. It is known that a six-digit palindrome is divisible by 75, and the quotient is still a palindrome. Find the quotient.

26. From 2021 to 8999, how many integers have its sum of digits divisible by 7?

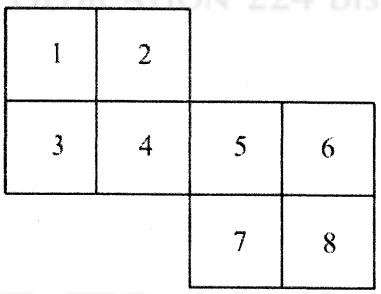
27. Six distinct numbers a, b, c, d, e and f are 1, 2, 3, 4, 5 and 6 in some order. The sum of the three fractions $\frac{a}{b}+\frac{c}{d}+\frac{e}{f}$ is calculated. If the smallest possible sum is $\frac{x}{y}$ in its simplest form, what is x+y?

28. In the diagram below, the rectangle and the square have their vertices on that of the two identical regular 12-sided regular polygon. If the ratio of the area of the rectangle to that of the square is a:b in its simplest form, find the value of a+b.



29. On an infinitely large flat surface, there are n straight lines. Each line intersects exactly 3 other lines, and no three lines meet at a common point. Find the value of n.

30. Eight squares below are labelled from 1 to 8. Two of the squares are to be removed so that the remaining 6 squares form the net of a cube. In how many different ways can this be done? Note: A net of a cube made of 6 squares such that when the net is folded along the grid lines, a cube can be formed.



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