Mathlete Training Centre
Round 2 RIPMWC Open

2015 RIPMWC Open Round 2 Answers

1) $\left[10\frac{1}{20}+(3-0.85)\div\frac{5}{6}\right]\div 505.2$
$=\left[10.05+2.15\times\frac{6}{5}\right]\div(40\times 12.63)$
$=12.63\div(40\times 12.63)=0.025$

2) $12090=2\times 3\times 5\times 13\times 31$
a and b must share the remaining 4 factors, 2, 3,5 and 13 in order for the 2 conditions to be satisfied.
No. of ways $=1+4+6+4+1=2^4=16$

3) $1\frac{1}{10}+4\frac{1}{40}+7\frac{1}{88}+10\frac{1}{154}+13\frac{1}{238}+16\frac{1}{340}$
$=1\frac{1}{2\times 5}+4\frac{1}{5\times 8}+7\frac{1}{8\times 11}+10\frac{1}{11\times 14}+13\frac{1}{14\times 17}+16\frac{1}{17\times 20}$
$=(1+4+7+10+13+16)+\frac{1}{3}\times\left(\frac{5-2}{2\times 5}+\frac{8-5}{5\times 8}+\frac{11-8}{8\times 11}+\frac{14-11}{11\times 14}+\frac{17-14}{14\times 17}+\frac{20-17}{17\times 20}\right)$
$=(1+4+7+10+13+16)+\frac{1}{3}\times\left(\frac{1}{2}-\frac{1}{5}+\frac{1}{5}-\frac{1}{8}+\frac{1}{8}-\frac{1}{11}+\frac{1}{11}-\frac{1}{14}+\frac{1}{14}-\frac{1}{17}+\frac{1}{17}-\frac{1}{20}\right)$
$=(1+4+7+10+13+16)+\frac{1}{3}\left(\frac{1}{2}-\frac{1}{20}\right)$
$=3\times 17+\frac{1}{3}\times\frac{9}{20}=51\frac{3}{20}$

4) $\frac{1}{2015^3-2014\times(2015^2+2016)}=\frac{1}{2015^2(2015-2014)-(2015-1)(2015+1)}=\frac{1}{2015^2-(2015^2-1)}$
$=1$

5) Note that since each letter is first digit at least one number, none of the letters represent the digit zero. From the ones column of the addition, it follows that the number 7B ends in the digit A which implies that A and B are either both even or both odd. Since the top row of the addition is clearly smaller than the bottom row, the digit A must be greater than the digit B. Because there is no carry from the hundred-thousands place to the millions place in the addition, B must be less than 5, and A must be at least 2B.

Sicne the carry into the hundred-thousands place is at most 3, A cannot exceed 2B+3. The possible values for (B, A) that satisfy $2B \leqslant a \leqslant 2B + 3$, $B < 5$, with A and B having the same parity are (1, 3), (1, 5), (2, 4), (2, 6), (3, 7), (3, 9), (4, 8). But the ones column shows that 7B must end in the digit A. This limits the possibilities to (2, 4) and (4,8). If B=2, then there is a carry of 1 from the ones column to the tens column. But then 6D + 2 must exceed 10 and end in the digit 8 which requires D to be 6.
Thus there will be a carry of 3 from the tens column to the hundred column, so 5E+ 3 ends in the digit 8 implying that E is odd. Moreoever, the thousands column contains four 6's and thus, with the carry, must add to 28. The carry from the hundreds column must be 4, implying that E=9. Finally, the ten-thousands column has 3C + 2 adding to 8, so C=2. This determines that ABCDE = 84269.

6) No. of blue marbles removed $= \dfrac{5}{5+9} \times 98 = 35$
No. green marbles removed = 98 - 35 = 63
Let x and y be the number of remaning blue and green marbles respectively.
$x = \frac{2}{7}y$
$x + 35 = \frac{3}{7}(y + 63)$
$\frac{2}{7}y + 35 = \frac{3}{7}y + 27$
$\frac{1}{7}y = 8$
y= 56
x=16
There total number of marbles = 98 + 56 + 16 = 170

7) Let 2C be the circumference. The ratio of the distances travelled by Barry and Ivan remains constant because their speeds are uniform. Therefore
$\dfrac{C-90}{90} = \dfrac{2C-50}{C+50}$
$\rightarrow C^2 - 220C = 0$
$\rightarrow C = 220$
$\rightarrow$ Circumference is 440m

8) $6 \equiv 6\ (mod\ 100)$
$6^2 \equiv 36 (mod 100)$
$6^3 \equiv 16 (mod 100)$
$6^4 \equiv 96 (mod 100)$
$6^5 \equiv 76 (mod 100)$
$6^6 \equiv 56 (mod 100)$
$6^7 \equiv 36 (mod 100)$
$15 \equiv 15\ (mod\ 100)$
$15^2 \equiv 25 (mod 100)$
$15^3 \equiv 75 (mod 100)$
$15^4 \equiv 25 (mod 100)$
...
2015 - 1 = 402 $\times 5 + 3$
Last 2 digits of 6^{2015} is 76
Last 2 digits of 2015^6 is 25.
Last 2 digit 2015×6 is 90
Last 2 digit of the sum is 91.

9) 2 "0" s: 9
$1"0" : 2 \times 9 = 18$
$No"0" : 9 \times 8 \times \frac{3!}{2!} = 216$
Total = 9 + 18 + 216 = 243

10) No. of car registation number = $({}^4C_2 \times 4!) \times ({}^6C_1 \times 2!)$
= 1728

11) $21 \times 19 + 1 = 400, 401, ..., 22 \times 19 + 1 = 419$
$21 \times 20 + 1 = 421, 422, ..., 22 \times 20 + 1 = 441$

12) $\frac{1}{3^7+1} + \frac{1}{3^7+3} + \frac{1}{3^7+3^2} = \frac{1}{3^7+3^3} + ... + \frac{1}{3^7+3^{13}} + \frac{1}{3^7+3^{14}}$
$= (\frac{1}{3^7+1} + \frac{1}{3^7+3^{14}}) + (\frac{1}{3^7+3} + \frac{1}{3^7+3^{13}}) + ... + (\frac{1}{3^7+3^6} + \frac{1}{3^7+3^8}) + \frac{1}{2(3^7)}$

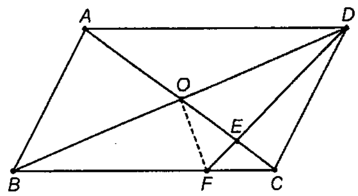
13) Let the length of each side of the larger square be a cm
Area of the triangle AED = $(a^2 + 18^2) - \frac{1}{2}a(a-18) - \frac{1}{2}a(a+18) - \frac{1}{2}(18)^2$
$= \frac{1}{2}(18)^2$

=162 cm sq

14) Let x be the number of packets of salt sold and y be the number of packets of sugar sold.
5x + 9y = 2015
x+ y < 350
It follows that

$$x = \frac{2015-9y}{5} = \frac{2015-10y+y}{5} = (403-2y) + \frac{y}{5}$$
$$2015 = 5(x+y) + 4y < 5 \times 350 + 4y$$
$$y > 66\frac{1}{4}$$
Since y is also divisible by 5, minimum value of y is 70.

15) Let Area (CEF) = x, Area (CED) = y, Area(DEO) = z and Area (OEF) = w



$$y + z = \frac{1}{4}$$
$$x + y = \frac{1}{8}$$
$$x + w = \frac{1}{16}$$
$$\frac{x}{y} = \frac{w}{z} \to \frac{x}{\frac{1}{8}-x} = \frac{\frac{1}{16}-x}{\frac{1}{8}+x} \to x = \frac{1}{40}$$
Note: a simpler solution uses areas of similar triangles
Area(AED)= $4^2\times$ Area (CEF)
Area (EDC) = $4 \times$ Area(CEF)
Area (CEF) = $\frac{1}{4^2+4}$ Area (ACD)
$= \frac{1}{20} \times \frac{1}{2} = \frac{1}{40}$