Mathlete Training Centre
2014   Open Round 2

RIPMWC

1) The fraction $\dfrac{2014}{37}$ can be written in the form $54 + \cfrac{1}{x + \cfrac{1}{y + \cfrac{1}{z}}}$, where x, y, z are distinct integers. Find the value of $x + y + z$

2) Two workers A and B work at a constant rate to complete a task. If A works on the task for 15 minutes and then B takes over, the task will be completed in another 9 minutes. If B works on the task first for 27 minutes and then A takes over, the task will be completed in another 9 minutes. Now, A starts working on the task first for 6 minutes and B takes over, how many minutes does B need to work so that the task will be completed?

3) Calculate the value of $(3999 \times 2014.2014 + 3999.3999 \times 2014) \div 12.0012$.

4) How many different ways are there to form a three-digit even number choosing the digits from 0, 1, 2, 3, 4, 5 and 8 without repetition?

5) Jack and Jill play a game of picking up coins from a pile of 2014 coins. They take turns altenately with Jack starting first. In each turn, Jack and Jill can pick up 1, 2, 3, 4, 5 or 6 coins. The one who takes the last coin is the loser. How many coins shoud Jack pick up in the first turn to ensure that he will be the winner?

6) How many different numbers can be obtained by adding three distinct numbers chosen from the set {2014, 2017, 2020, 2023, ..., 2059, 2062}.

7) There are 2 containers A and B. Container A contains $8060cm^3$ of milk and container B is empty.

The first pour is to pour $\frac{1}{2}$ of the milk in container A into container B.

The second pour is to pour $\frac{1}{3}$ of the milk in container B into container A.

The third pour is to pour $\frac{1}{4}$ of the milk in container A into container B.

The fourth pour is to pour $\frac{1}{5}$ of the milk in container B into container A.

Find the amount of milk in container A after the 2014th pour in cm^3.

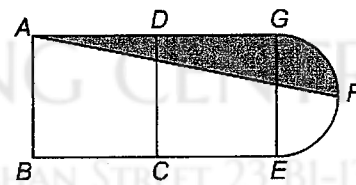8) In the figure, ABCD and DCEG are identical squares. The point F is the mid-point of the semicircle EFG. If the perimeter of ABCEFGDA is $26\frac{2}{7}$cm, calculate the area of the shaded part in cm^2.

(Assume $\pi = \frac{22}{7}$)

9) If $N = 1141024102...41028448$, which is n times of 4102, is divisible by 264, find the largest possible value of n which is less than 300.

10) In how many ways can 6 students A, B, C, D, E, F line up in a row if students A and B are always next to each other?

11) From a point A on the circumference of a circle, moving along the circumference of the circle, John made a pencil mark every 36cm until the last pencil mark at A after completing one round. From point A again, moving along the circumference of the circle, he made a pencil mark every 42cm until last pencil mark again at A after completing the second round. Note that some of the pencil marks coincided. Given that there are altogether 48 distinct pencil marks, find the circumference of the circle in cm.

12) There are 11 identical chocolate bars in a jar. Gopal can only eat 1 or 2 of these chocolate bars at a time. He does this until there are no more chocolate bars left. In how many different ways can he do this?

13) The diagram shows a △AEG.
    B, C, and D are points on AE such that AB = BC = CD = DE.
    J, I, and h are points on AG such that AJ = JI = IH = HG.
    F is a point on EG such that $EF : FG = 3 : 1$.
    Given that the area of trapezium BCIJ is $60cm^2$,
    find the area of trapezium FGHP is cm^2.
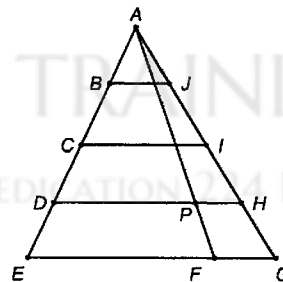
14) Calculate $\dfrac{1}{53} + \dfrac{1}{53 + 106} + \dfrac{1}{53 + 106 + 159} + ... + \dfrac{1}{53 + 106 + 159 + ... + 1961 + 2014}$.

15) What are the last 2 digits of $2^{3^{2014}}$?