Mathlete Training Centre
Round 2 RIPMWC Open

2013 RIPMWC Open Round 2 Answers

1) $2^3 + 2^4 + 2^5 + ... + 2^{11} + 2^{12} = 2^3(1 + 2 + 2^2 + ... + 2^8 + 2^9) = 8 \times \dfrac{2^{10} - 1}{2 - 1} = 8184$

2) $\dfrac{122 + 183 + 244 + 305 + ... + 1952 + 2013}{44 + 48 + 52 + ... + 280 + 284}$

$= \dfrac{61(2 + 3 + 4 + ... + 33)}{4(11 + 12 + 13 + ... + 71)}$

$= \dfrac{61(35)\left(\dfrac{32}{2}\right)}{4(82)\left(\dfrac{61}{2}\right)}$

$= \dfrac{140}{41} = 3\dfrac{17}{41}$

3) $1^{2013} = 1$ so tens digit is 9
$1111 \equiv 111 \equiv 11 (mod 100)$
$11^2 \equiv 21 \pmod{100}$
$11^3 \equiv 31 \pmod{100}$
$11^4 \equiv 41 \pmod{100}$
$11^5 \equiv 51 \pmod{100}$
$11^6 \equiv 61 \pmod{100}$
$11^7 \equiv 71 \pmod{100}$
$11^8 \equiv 81 \pmod{100}$
$11^9 \equiv 91 \pmod{100}$
$11^{10} \equiv 1 \pmod{100}$
$1^{2013} + 11^{2013} + 111^{2013} + 1111^{2013} \equiv 31 + 31 + 31 + 1 \equiv 94 \pmod{100}$

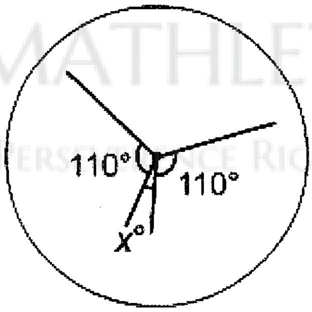
4) All the switches start in the off position.
If a switch is changed to an odd number of times then it will finish in the on position.
If it is changed an even number of times then it will finish in the off position.
Each switch is changed as many times as the number on it has factors.
Perfect square numbers have an odd number of factors but all other numbers have an even number of factors.
So the switches numbered with perfect squares will be on at the end of the process and all the other switches will be off.
There are 44 perfect squares less than or equal to 2013 so 44 switches will be on.
$2013 - 44 = 1969$ switches will be off.

5) $(x \times 2 + 3) \times \frac{4}{3} + (8 - 4) \times 48 = 4x$

$(2x + 3) \times \frac{4}{3} + 4 \times 48 = 4x$

$x = 147$

6) From the diagram



Let $x°$ be the amount the hour hand moved during the interval.

In the same interval, the minute hand moves $\frac{x°}{30°} \times 360° = 12x°$.

$220 + x° = 12x°$

$x = 20$

Minute hand moves 240°.

Time interval $= \frac{240}{6} = 40$ min

7) $5 + 10 + 15 + ... + 5n = 5(1 + 2 + 3 + ... + n) = \frac{5n(n+1)}{2}$

$2013 = 3 \times 11 \times 61$

Either n or $n + 1$ is a multiple of 61 and $n(n + 1)$ is divisible by 3, 11 and 61.

60, 61, 62 cannot

121, 122, 123 cannot

182, 183, 184 cannot

243, 244, 245 cannot

304, 305, 306 cannot

365, 366, 367 cannot

426,427,428 cannot

487,488,489 cannot

548, **549,550** Yes

So least n is 549.

8) Let a_n be the number of such arrangements for n students.
$a_n = a_{n-1} + a_{n-2}, n \geq 3$
$a_1 = 1, a_2 = 2$
$a_3 = 3$
$a_4 = 5$
$a_5 = 8$
$a_6 = 13$
$a_7 = 21$
$a_8 = 34$
$a_9 = 55$
$a_{10} = 89$
$a_{11} = 144$
$a_{12} = 233$

9) $$\frac{(2013^2-2011^2)(2013^2-2010^2)...(2013^2-1^2)(2013^2-0^2)}{(2012^2-2011^2)(2012^2-2010^2)...(2012^2-1^2)(2012^2-0^2)}$$
$$=\frac{(4024\times 2)\times(4023\times 3)\times ...\times(2014\times 2012)\times(2013\times 2013)}{(4023\times 1)\times(4022\times 2)\times ...\times(2013\times 2011)\times(2012\times 2012)}$$
$$=\frac{4024\times 2013}{2012}=4026$$

10) 1-digit $= 5$
2-digit $= 5\times 8 = 40$
3-digit $= 5\times 1\times 8 + 5\times 8\times 7 = 320$
4-digit $= 1\times 8\times 7\times 4 + 1 = 1225$
Total $= 5 + 40 + 320 + 1225 = 590$

11) $$\frac{49+\frac{7}{8}(m-52)}{m}\geq\frac{9}{10}$$
$$49+\frac{7}{8}m-\frac{91}{2}\geq\frac{9}{10}m$$
$$\frac{m}{40}\leq\frac{7}{2}$$
$$m\leq 140$$

12) Shaded area $= \left[\frac{30°}{360°}(2\pi\times 8^2)-3\times 8\right]-\left[3\times 8-\frac{30°}{360°}(2\pi\times 6^2)\right]$
$$=\frac{\pi}{6}(8^2+6^2)-48$$
$$=\frac{50}{3}\times\frac{22}{7}-48$$
$$=4\frac{8}{21}$$

13) Area of $BRD : BRC = 2 : 1$
Area of $CRD : BRD = 2 : 1$
Let area of BRC be a.
Area of $BRD = 2a$ and area of $CRD = 4a$
Area of $CPR = \frac{2}{3}a$
Area of $ABRD$: Area of CPR = 9a $: \frac{2}{3}a = 27 : 2$

14) Triangle URS and triangle TRQ are identical (congruent) triangles.
Hence $RU = RT$
$\frac{1}{2}(RU)(RT) = \frac{1}{2}(RT)^2 = 212.5$
$(RT)^2 = 425$
$(RQ)^2 = 256$
By Pythagoras' theorem, $(QT)^2 = 425 - 256 = 169$
$QT = 13$

15) Let a, b and c be the time in hours taken by Ali, Brian and Chandru respectively to finish the job.
$\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = \frac{1}{a-6} = \frac{1}{b-1} = \frac{1}{\frac{c}{2}} = \frac{2}{c}$
Let $\frac{1}{a} + \frac{1}{b} = \frac{1}{h}$
$\frac{1}{h} + \frac{1}{c} = \frac{2}{c} \Rightarrow h = c$
$\frac{1}{a-6} = \frac{1}{h} + \frac{1}{h} = \frac{2}{h} \Rightarrow \text{a} = \frac{h+12}{2}$
$\frac{1}{b-1} = \frac{2}{h} \Rightarrow b = \frac{h+2}{2}$
$\frac{2}{h+12} + \frac{2}{h+2} = \frac{1}{h} \because \frac{1}{a} + \frac{1}{b} = \frac{1}{h}$

$h(4h+28) = h^2 + 14h + 24$
$3h^2 + 14h - 24 = 0$
$(3h-4)(h+6) = 0$
$h = 1\frac{1}{3}$