

Mathlete Training Centre
Round 2 RIPMWC open

2013 RIPMWC open round 2

1.

Find the sum $2^3 + 2^4 + 2^5 + \dots + 2^{11} + 2^{12}$.

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2.

Evaluate $\frac{122 + 183 + 244 + 305 + \dots + 1952 + 2013}{44 + 48 + 52 + \dots + 280 + 284}$

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3. Find the remainder when $1^{2013} + 11^{2013} + 111^{2013} + 1111^{2013}$ is divided by 100.

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4. 2013 switches are numbered from 1 to 2013. They are all switched off initially. All the switches numbered with a multiple of 1 are changed from off to on. Next the states of all switches which are numbered with a multiple of 2 are changed (that is, the ones that are off are turned on, the ones that are on are turned off). Then the states of all the switches numbered with a multiple of 3 are changed. This continues, until finally the states of all the switches with a multiple of 2013 are changed. How many switches are off at the end of this process?

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5.

In the first week of 2012, Raffles Institution Mathematics Club had x members, the members had so much fun during the training in the 1st week that in the second week each of the members brought a new member with them. In the third week, 3 more students joined and in the fourth week the number of members increased by $\frac{1}{3}$. In each the following weeks after the fourth week, 8 more students joined the club and 4 students dropped out. The number of members in the club in the 52th week was 4 times the initial number. Find x .

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6.

At the start of his dinner after 6 pm, Ivan observes that the hands of his watch form an angle of 110° . When he finishes his dinner before 7 pm, he notices that the hands of his watch forms an angle of 110° . Find the number of minutes that he takes to finish his dinner.

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7. If n is a positive integer such that $5 + 10 + 15 \dots + 5n$ is divisible by 2013. Find the smallest possible value of n .

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8. 12 boys are sitting on 12 chairs fixed to the ground for their formal class photo. The chairs are numbered as Seat 1 to 12 from left to right. For their informal class photo, the boys are allowed to change their seats. He can either choose to remain in his original seat or change to an adjacent seat. For example, a boy originally in Seat 4 can choose to remain in his original seat or move to Seat 3 or 5. For the boy originally in Seat 1 or 12, he can choose to remain in his original seat or move to Seat 2 or 11 respectively. If no 2 boys can share the same seat, how many arrangements are there for the informal class photo?

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9.

Evaluate:
$$\frac{(2013^2 - 2011^2)(2013^2 - 2010^2)\dots(2013^2 - 1^2)(2013^2 - 0^2)}{(2012^2 - 2011^2)(2012^2 - 2010^2)\dots(2012^2 - 1^2)(2012^2 - 0^2)}$$

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10.

2013 is an odd number and all its digits are different. Among all the positive integers from 1 to 2013, how many odd numbers are there with all its digits different?

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11.

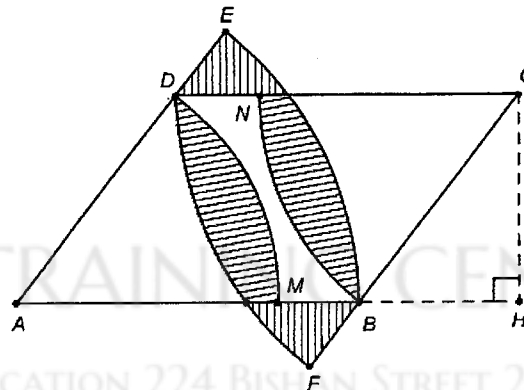
In counting m coloured balls, some green and some blue, it was found that 49 of the first 52 counted were green. Thereafter, 7 out of 8 counted were green. Given that at least 90% of the balls were green, find the largest possible value of m .

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12.

$ABCD$ is a parallelogram with $\angle DAB = 30^\circ$, $AD = 6$ cm, $AB = 8$ cm and height $CH = 3$ cm. Arc BE and arc DF are drawn using AB and CD as radius respectively and arc DM and arc BN are drawn using AD and CB as radius respectively. By taking $\pi = \frac{22}{7}$, find the area of the shaded region in cm^2 .

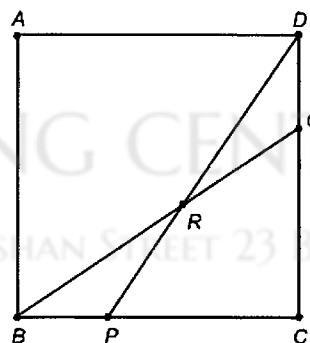


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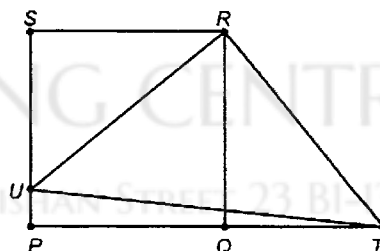
13.

In the figure, $ABCD$ is a square. P and Q are points on BC and CD respectively such that $BP = CQ = \frac{1}{3} CD$. BQ and DP intersect at R . Find the ratio of area of quadrilateral $ABRD$ to area of $\triangle CPR$.



14.

As shown in the diagram above, $PQRS$ is a square, U is a point on PS and T is on PQ extended such that $\angle URT = 90^\circ$ and RT meets PQ extended at T . The area of square $PQRS$ is 256 cm^2 and the area of $\triangle URT$ is 212.5 cm^2 . Find the length QT in cm.



15. 3 workers Ali, Brian and Chandru, working together, finish a job 6 hours faster than Ali would take alone, 1 hour faster than Brian would take alone and half the time needed by Chandru when working alone. Find the time in hours for Ali and Brian working together to finish the job.

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