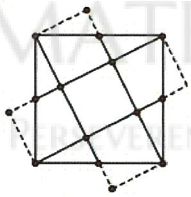
Mathlete Training Centre
Round 2 RIPMWC Open

2012 RIPMWC Open Round 2 Answers

1) From the diagram,



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By rotating the 4 smaller triangles into the position as shown, 4 additional squares are formed with the same area as that in the centre.
Area of the smaller square is $\frac{1}{5} \times 12 = 2.4$ units^2

2) $242 \equiv 2 \pmod 4 \Rightarrow$ last digit of 13^{242} is 9
$379 \equiv 3 \pmod 4 \Rightarrow$ last digit of 17^{379} is 3
Last digit of $4^{2012} = 6$
Last digit of $13^{242} + 17^{379} \times 4^{2012}$ is 7

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3) $$\frac{(9 \times 10) + 2}{9 \times 10} + \frac{2(10 \times 11) + 2}{10 \times 11} + \frac{3(11 \times 12) + 2}{11 \times 12} + ... + \frac{51(59 \times 60) + 2}{59 \times 60}$$
$$= (1 + 2 + 3 + ... + 51) + 2 \times \left(\frac{1}{9} - \frac{1}{10} + \frac{1}{10} - \frac{1}{11} + ... + \frac{1}{59} - \frac{1}{60}\right)$$
$$= \frac{51 \times 52}{2} + 2 \times \left(\frac{1}{9} - \frac{1}{60}\right)$$
$$= 1326\frac{17}{90}$$

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4) Let the lengths of its sides be a, b and c and $a \leq b \leq c$.

If $c = 13$, $(a, b) = (1, 13), (2, 12), (3, 11), ..., (7, 7)$. Total $= 7$

If $c = 12$, $(a, b) = (3, 12), (4, 11), (5, 10), (6, 9), (7, 8)$. Total $= 5$

If $c = 11$, $(a, b) = (5, 11), (6, 10), (7, 9), (8, 8)$. Total $= 4$

If $c = 10$, $(a, b) = (7, 10), (8, 9)$. Total $= 2$

If $c = 9$, $(a, b) = (9, 9)$. Total $= 1$

Total number of triangles $= 19$

5) $\dfrac{1}{x}+\dfrac{1}{y}=\dfrac{1}{2012}$

$\dfrac{x+y}{xy}=\dfrac{1}{2012}\Rightarrow 2012x+2012y=xy\ x=\dfrac{2012y}{y-2012}$

Greatest value of x is when $y-2012=1$

Greatest x $=2012\times 2013=(2000+12)(2000+13)=4050156$

6) $\dfrac{(2009^2-2009-6)(2009^2+2\times 2009-3)}{2006\times 2008\times 2010\times 2011\times 2012}$

$=\dfrac{(2009-3)(2009+2)(2009+3)(2009-1)}{2006\times 2008\times 2010\times 2011\times 2012}$

$=\dfrac{1}{2010}$

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7) **Method 1 (Work Backward)**

	Player 1	Player 2	Player 3
After 3rd game	24	24	24
After 2nd game	12	12	48
After 1st game	6	42	24
Initial	39	21	12

Method 2

Let x be the no of points that Player 1 starts with and $72-x$ be the total points that Players 2 and 3 started with.

After game 1, Player 1 has $x-(72-x)=2x-72$

After game 2, Player 1 has $2(2x-72)$

After game 3, Player 1 has $4(2x-72)$

Hence, $4(2x-72)=24$

$x-36=3$

$\Rightarrow x=39$

Let Player 2 start with y points. Then Player 3 starts with $33-y$.

After game 1, Player 2 has $2y$ and Player 3 has $2(33-y)$

After game 2, Player 2 has $2y-(6+66-2y)=4y-72$

After game 2, Player 2 has $2(4y-72)=24$

$y-18=3$

$y=21$

Player 3 starts with $33-21=12$

8) $396 = 2^2 \times 3^2 \times 11$
Since R ends with 12, it is divisible by 4.
As the sum of digits of "992012" is 23 which is not divisible by 9, k must be divisible by 9.
Difference of sum of digits in even place and sum of digits in odd place $= 9-9+2-0+1-2 = 1$
For R to be divisible by 11, k must be divisible by 11.
Hence the smallest possible value of $k = 9 \times 11 = 99$

9) Let w be the width of the river. When they first meet, Ferry 1 has travelled 900m, while Ferry 2 has travelled $(w-900)$m.
When they meet for the second time,
Ferry 1 has travelled an additional $(w-900)+500 = (w-400)m$
Ferry 2 has travelled an additional $900+(w-500) = (w+400)m$
Since each ferry is travelling at constant speed,
$\dfrac{900}{w-900} = \dfrac{w-400}{w+400}$
$(w-400)(w-900) = 900(w+400)$
$w^2 - 2200w = 0$
$w = 2200$

10) To find the largest n such that
$14n \leq 10^{2012} - 1$
$\Longleftrightarrow 14n < 10^{2012}$
$\Longleftrightarrow n < \dfrac{5}{7} \times 10^{2011}$
This is equivalent to calculating $\dfrac{5}{7} \times 10^{2011}$ and round down to the nearest integer.
or just calculating $\dfrac{5}{7} \times 10^{2011}$ and truncating the number at the decimal point.
Since $\dfrac{5}{7} = \overline{0.714285}$ and $2011 = 335 \times 6 + 1$
$\therefore n = \underbrace{714285...714285}_{335 \text{ times}}7$
Sum of digits $= 27 \times 335 + 7 = 9052$

11) The decimal expansion converts to a fraction with denominator 99.
Of the 99 possible values for the numerator, we first remove those not relatively prime to 99.
These are:
1. 33 multiples of 3
2. 9 multiples of 11
3. 3 multiples of 33

Number of irreductible fractions with denominator 99 is $99-33-9+3 = 60$— All the reducible fractions do not produce new values for the numerator, except those whose numerator is a multiple of 27, namely, 27, 54 and 81 and the corresponding reduced fractions are $\dfrac{3}{11}, \dfrac{6}{11}$ and $\dfrac{9}{11}$.
Hence the total number of possible values of $A = 60 + 3 = 63$

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12) Consider 3 cases:

(a) Numbers from 1 to 999, $\overline{abc}$ and $a+b+c=10$

Number of integers $=\binom{10+3-1}{3-1}-3=\binom{12}{2}-3=63$ (Note: must exclude $0+0+10$ etc.)

(b) Numbers from 1000 to 1999, $\overline{abc}$ and $a+b+c=9$

Number of integers $=\binom{9+3-1}{3-1}=\binom{11}{2}=55$

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(c) Number from 2000 to 2011, number of such integers $=1$

Total number of such integers $=63+55+1=119$

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13) Let x be the number of bottles of lemonade sold and y be the number of bottles of 1000 Plus sold.

$4x+7y=2012$

$x+y<350$

It follows that

$x=\dfrac{2012-7y}{4}=\dfrac{2012-8y+y}{4}=(503-2y)+\dfrac{y}{4}$

$2012=4(x+y)+3y<4\times 350+3y$

$y>204$

Since y is also divisible by 4, minimum value of y is 208.

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14) From the table below:

1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18
19	20	21	22	23	24	25	26	27
28	29	30	31	32	33	34	35	36
37	38	39	40	41	42	43	44	45
46	47	48	49	50	51	52	53	54
55	56	57	58	59	60	61	62	63
64	65	66	67	68	69	70	71	72

Those in yellow are removed from the list in the first round and those in green are removed from the list in the second round.
The remaining students after the first 2 rounds

1	2	5	7	10	11	14	16	19
20	23	25	28	29	32	34	37	38
41	43	46	47	50	52	55	56	59
61	64	65	68	70				

Round 3 Round 6
Round 4 Round 7
Round 5 Round 8
Only 7 and 55 left

Hence the teacher must have started counting at $64 - 55 + 1 = 10$

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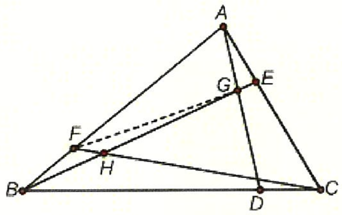
15) From the diagram



$$S_{\triangle BDE} = \frac{4}{5} S_{\triangle BCE} = \frac{4}{5} \times \frac{2}{3} = \frac{8}{15}$$

$$S_{\triangle ABE} = \frac{1}{3} \Rightarrow \frac{AG}{GD} = \frac{S_{\triangle ABE}}{S_{\triangle BDE}} = \frac{\frac{1}{3}}{\frac{8}{15}} = \frac{5}{8}$$

$$S_{\triangle AEG} = \frac{5}{13} S_{\triangle ADE} = \frac{5}{13} \times \frac{1}{3} \times S_{\triangle ACD} = \frac{5}{13} \times \frac{1}{3} \times \frac{1}{5} = \frac{1}{39}$$

$$\frac{FH}{HC} = \frac{S_{\triangle BEF}}{S_{\triangle BCE}} = \frac{\frac{1}{3} \times \frac{1}{4}}{\frac{2}{3}} = \frac{1}{8}$$

$$S_{\triangle BFH} = \frac{FH}{FC} \times S_{\triangle BCF} = \frac{1}{9} \times \frac{1}{4} = \frac{1}{36}$$

$$S_{AFGH} = \frac{1}{3} - \frac{1}{39} - \frac{1}{36} = \frac{131}{468}$$

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