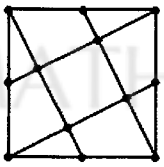
Mathlete Training Centre
Round 2 RIPMWC open

2012 RIPMWC open round 2

1.



The 4 corners of a square are joined to the mid-point of another side, as shown in the figure above, in the process forming a smaller square in the middle. If the area of the bigger square is 12 units^2, find the area of the smaller square in units^2 ?

2.

Find the unit digit of $13^{242} + 17^{379} \times 4^{2012}$.

3.

Calculate $\frac{(9\times10)+2}{9\times10}+\frac{2(10\times11)+2}{10\times11}+\frac{3(11\times12)+2}{11\times12}+\ldots+\frac{51(59\times60)+2}{59\times60}$.

4.

A triangle is such that the lengths of all of its sides are whole numbers and the perimeter is 27. How many such triangles are possible?

5.

If x and y are positive integers such that $\frac{1}{x}+\frac{1}{y}=\frac{1}{2012}$, find the greatest possible value of x.

6.

Calculate $\frac{(2009^2-2009-6)(2009^2+2\times2009-3)}{2006\times2008\times2010\times2011\times2012}$.

7.

Three persons play a game together; in the first game, the first person loses to each of the other two as many points as each of them has (For example, if among the other 2 persons, one has 6 points and the other has 8 points, then the first person loses 6 points and 8 points to each of them respectively). In the second game, the second person loses to each of the other two as many points they have already. In the third game, the first and second person gain from the third as many points as they had (before the third game). After the third game, they all have 24 points each. With how many points had each sat down to play?

8.

An integer R is formed by repeating "992012" k times, that is, $\underbrace{992012992012...992012}_{992012 \text{ repeated } k \text{ times}}$

If R is divisible by 396, find the smallest possible value of k.

9.

Two ferryboats ply back and forth across a river with constant but different speeds, turning at the riverbanks without loss of time. They leave opposite shores at the same instant, meet for the first time 900 metres from one shore, and meet for the second time 500 metres from the opposite shore. Find the width of the river?

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10.

Let n be the largest integer for which $14n$ has exactly 2012 digits. What is the sum of digits of n?

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11.

The fraction $\frac{A}{B}$ is in the lowest form. Its decimal expansion has the form $0.xyxyxyxy\ldots$.

The digits x and y may be equal, except that not both can be 0. Find the number of different values of A.

12.

Find the number of positive integers less than 2012 such that the sum of digits is 10.

13.

Ronald sold drinks at a sports match. He sold bottles of lemonade at $4 each and bottles of "1000 Plus" at $7 each. He started with a total of 350 bottles. Not all were sold and his total income was $2012. What was the minimum number of bottles of "1000 Plus" that Ronald could have sold?

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14.

A teacher arranges 72 students in a circle and numbers them 1 to 72 in clockwise order. The teacher counts them "One, Two, Three!" in clockwise order. On the count of three, the teacher removes the student and counts "One, Two, Three!" starting with the next student. As the teacher continues, the circle gets smaller and smaller until 2 students left. If the student with the higher number is 64, what is the number of the student from which the teacher starts counting?

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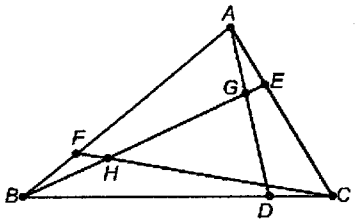
15.



In ΔABC, the point E is on AC such that $AE:EC = 1:2$, the point D is on CB such that $CD:DB = 1:4$ and the point F is on BA such that $BF:FA = 1:3$. Lines AD and BE intersect at point G. Lines BE and CF intersect at point H. Given that the area of $\Delta ABC = 1$ unit^2, find the area of quadrilateral $AFHG$ in unit^2.