

Mathlete Training Centre
Round 2 RIPMWC Open

2011 RIPMWC Open Round 2 Answers

1) Number of such numbers = $\frac{2011}{15} - \frac{2011}{60} = 134 - 33 = 101$

2) $1 + 111 + 11111 + 1111111 + \dots + \underbrace{111\dots111}_{2011 \text{ 1s}}$

There are $\frac{2011 + 1}{2} = 1006$ terms

$$\begin{array}{r} 1006 \\ 10050 \\ 100500 \\ 1004000 \\ + 10040000 = \dots \mathbf{55556} \end{array}$$

3) $x + y = 7(x - y) \Rightarrow 8y = 6x$
 $xy = 24(x - y) = 24x - 18x = 6x$
 $x(y - 6) = 0$
 $\Rightarrow y - 6 = 0$
 $y = 6$
 $\Rightarrow x = 8$
 $\therefore xy = 48$

4) Let the age of Mary's grandmother be $10x + y$.
Hence the age of Mary's mother is $10y + x$. $(10x + y) - (10y + x) > 0$
 $9(x - y) > 0$
 $0 < x - y < 9$
 $9(x - y) = 4z$ (given)
4 divides $(x - y)$
 $x - y = 4$ or 8

Case 1: $x - y = 8$, $x = 9$, $y = 1$, $z = 18 \Rightarrow$ the ages of Mary, her mother and grandmother are respectively 18, 19 and 91 which is not possible.

Case 2: $x - y = 4$, $z = 9$. Possible ages for her grandmother and mother are respectively (62, 26), (73, 37), (84, 48), (95, 59).

Hence Mary's current age is 9.

- 5) After drawing 801 red balls, 801 yellow balls and 208 green balls, we need to draw 1 more ball to guarantee that we have at least 802 balls of the same color.
Hence the minimum number = $801 + 801 + 208 + 1 = 1811$

- 6) Let $x = 201120112011$.

So, $x + 2 = 201120112013$ and $x - 2 = 201120112009$.

$$\begin{aligned} & (201120112011 \times 201120112011) - (201120112013)(201120112009) \\ &= x^2 - (x + 2)(x - 2) \\ &= 4 \end{aligned}$$

- 7) Let the total number of pupils in all the 3 classes be S and $\$x$ be the amount that each pupil in class C will get.

$$\begin{aligned} \frac{42S}{2} + \frac{42S}{3} + \frac{42S}{x} &= S \\ \frac{105}{5} + \frac{98}{7} + \frac{x}{29} &= 1 \\ \frac{42}{x} &= 1 - \frac{35}{35} = \frac{6}{35} \\ \Rightarrow x &= 245 \end{aligned}$$

Each pupil in class C will get \$245.

- 8) Let $2C$ be the circumference. The ratio of the distances travelled by Ivan and Barry remains constant because their speeds are uniform.

$$\begin{aligned} \text{i.e., } \frac{C - 100}{100} &= \frac{2C - 60}{C + 60} \\ \Rightarrow C^2 - 240C &= 0 \\ \Rightarrow C &= 240 \end{aligned}$$

The circumference of the track is 480m.

- 9) From the table:

Column 1	Column 2	Column 3
1	2	3
4	5	6
7	8	9
10	11	

Observe that the sum of any 3 numbers from each column will be divisible by 3.

Also when one number is selected from each column, the sum is divisible by 3.

$$\text{No. of triplets} = \binom{3}{3} + \binom{4}{3} + \binom{4}{3} + \binom{4}{1} \times \binom{4}{1} \times \binom{4}{1} = 57$$

- 10) By adding up the numbers
 1001, 1111, 1221, ..., 1991,
 2002, 2112, 2222, ..., 2992,

$$\begin{aligned} \text{Sum} &= \text{Sum of thousands} + \text{Sum of hundreds} + \text{Sum of tens} + \text{Sum of units} \\ &= \left(\frac{90000 + 10000}{2} \right) \times 9 + 45(100) \times 9 + 45(10) \times 9 + \left(\frac{90 + 10}{2} \right) \times 9 = 495000 \end{aligned}$$

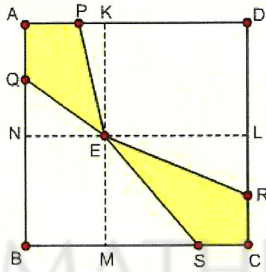
11)
$$\begin{aligned} &\frac{1}{3^6 + 1} + \frac{1}{3^6 + 3} + \frac{1}{3^6 + 3^2} + \frac{1}{3^6 + 3^3} + \dots + \frac{1}{3^6 + 3^{12}} \\ &= \left(\frac{1}{3^6 + 1} + \frac{1}{3^6 + 3^{12}} \right) + \left(\frac{1}{3^6 + 3} + \frac{1}{3^6 + 3^{11}} \right) + \dots + \left(\frac{1}{3^6 + 3^5} + \frac{1}{3^6 + 3^7} \right) + \frac{1}{3^6 + 3^6} \\ &= \frac{3^6 + 1}{3^6(3^6 + 1)} + \frac{3^5 + 1}{3^6(3^5 + 1)} + \dots + \frac{3 + 1}{3^6(3 + 1)} + \frac{1}{2(3^6)} \\ &= \frac{1}{3^6} + \dots + \frac{1}{3^6} + \frac{1}{2(3^6)} \\ &= \frac{13}{2} \times \frac{1}{3^6} = \frac{13}{1458} \end{aligned}$$

- 12) The various combinations can be summarised in the table below:

One Beep																	
Hr	Min	Hr	Min	Hr	Min	Hr	Min	Hr	Min	Hr	Min						
00	29	01	19	02	09	03	08	04	07	05	06						
	38		28		18		17		16		15						
	47		37		11		27		12		26	13	25	14	24		
	56		46		20		36		21		35	22	34	23	33		
			55				45				54		44		53		43
Hr	Min	Hr	Min	Hr	Min	Hr	Min	Hr	Min	Hr	Min						
06 15	05	07	04	08	03	09	02	10	01	11	00						
	14		13		12		11		10		09	08	07	06	05		
	23		22		17		21		18		20	19	18	17	16	15	
	32		31		30												
	41		40														
50																	
Two Beeps																	
Hr	Min	Hr	Min	Hr	Min	Hr	Min	Hr	Min	Hr	Min						
08	59	09	49	19	39												
17	18	58	19	48													
				57													

$$\begin{aligned} \text{Total number of beeps} &= 4 + 10 + 18 + 18 + 18 + 18 + 12 + 10 + 8 + 6 + 2 + 4 + 8 + 6 \\ &= 142 \end{aligned}$$

- 13) Let KM be the line segment through E parallel to AB with K on AD and M on BC .
 Let NL be the line segment through E parallel to AD with N on AB and L on DC .



Summing up areas of triangles APE , CES , CER and AQE .

$$\text{Total area} = \frac{1}{2} \times 2 \times (\text{KE} + \text{EM} + \text{LE} + \text{EN})$$

$$6 + 8 = \text{KM} + \text{LN}$$

$$14 = 2\text{AB}$$

$$\therefore \text{AB} = 7$$

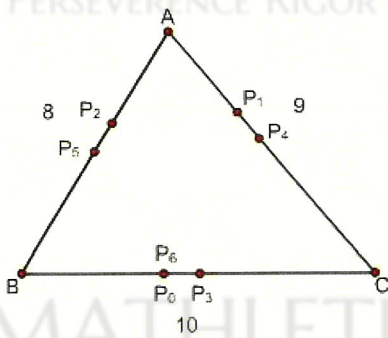
$$\text{Area of square} = 7 \times 7 = 49 \text{ cm}^2$$

- 14) Number of paths which does not pass through P

$$= \frac{10 \times 9 \times 8 \times 7 \times 6}{1 \times 2 \times 3 \times 4 \times 5} - \left(\frac{5 \times 4}{1 \times 2} \times \frac{5 \times 4}{1 \times 2} \right)$$

$$= 252 - 100 = 152$$

- 15) From the diagram,



$$BP_0 = 4 \Rightarrow CP_0 = 10 - 4 = 6$$

$$CP_1 = CP_0 = 6 \Rightarrow AP_1 = 9 - 6 = 3$$

$$AP_2 = AP_1 = 3 \Rightarrow BP_2 = 8 - 3 = 5$$

$$BP_3 = BP_2 = 5 \Rightarrow CP_3 = 10 - 5 = 5$$

$$CP_4 = CP_3 = 5 \Rightarrow AP_4 = 9 - 5 = 4$$

$$AP_5 = AP_4 = 4 \Rightarrow BP_5 = 8 - 4 = 4$$

$$BP_6 = BP_5 = 4 \Rightarrow CP_6 = 10 - 4 = 6$$

P_6 and P_0 coincides.

$$2011 = 6 \times 335 + 1$$

P_{2010} coincides with P_0

So P_{2011} is on AC such that $CP_{2011} = 6\text{cm}$

\therefore the shortest distance P_{2011} to $P_0 = 6 + 6 = 12\text{cm}$.