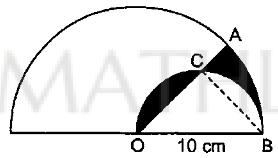
Mathlete Training Centre
Round 2 RIPMWC Open

## 2010 RIPMWC Open Round 2 Answers

1) Join BC.




Area of the segment OC = Area of the segment CB
So, shaded region = Area of sector OAB − Area of △ OCB
In △ OCB
$\angle OCB = 90°$, $\angle COB = \angle CBO = 45°$

$$\text{Area of shaded region} = \frac{45°}{360°} \times 10^2 - \frac{1}{2} \times 10 \times 5 = 14\frac{2}{7}\text{cm}^2$$

2) 8 types of coins will suffice
An example is 1, 2, 4, 8, 16, 32, 64 & 128.
Fewer coins cannot work since with n coins, there is at most $2^n$ combinations if each type of coin is used 0 or 1 time and $2^7 = 128 < 200$.

3) $$\frac{2010.201 + 20102010 + 201020102010 + 2010201020102010}{2012.2012 + 20122012 + 201220122012 + 2012201220122012}$$
$$= \frac{2010 \times (1.0001 + 100001 + 100010001 + 1000100010001)}{2012 \times (1.0001 + 10001 + 100010001 + 1000100010001}$$
$$= \frac{1005}{1006}$$

4) The largest 2-digit prime number is 97. Working backwards,
$97 + 192 = 289$
$\sqrt{289} = 17$
$17 \times 3 = 51$
$51 + 5 = 56$
$\frac{56}{2} = 28$
The answer is 28.

5) Let the time that would elapse be x minutes.
$\frac{x}{60} \times 360 - \frac{x}{60} \times 30 = 60$
$x = 10\frac{10}{11}$

6) There are $4!(=24)$, 4-digit numbers in which the digits 1, 2, 4, 5 appears exactly once.
These numbers can be paired such that the sum is $2145 + 4521 = 6666$.
The required sum is $6666 \times 12 = 79992$

7) Let the combined area of land be $5x$.
Area of John's plot = 3x
Area of Terry's plot = 2x
Total area of barley $= \frac{7}{2}x$
Total area of corn $= \frac{3}{2}x$
On John's land,
area of barley $= \frac{4}{5} \times 3x = \frac{12}{5}x$
area of corn $= \frac{1}{5} \times 3x = \frac{3}{5}x$

On Terry's land,
area of barley $= \frac{7}{2}x - \frac{12}{5}x = \frac{11}{10}x$
area of corn $= \frac{3}{2}x - \frac{3}{5}x = \frac{9}{10}x$



Hence, on Terry's land, barley : corn $= \frac{11}{10}x : \frac{9}{10}x = 11 : 9$

8) Let the number of soldiers be $n$.
$n = 3a + 1$, for any positive number a
$\Rightarrow 3a + 1 \equiv 2 \pmod 5$
$\Rightarrow a = 5b + 2$, where b is any positive number
$\Rightarrow n = 3(5b+2) + 2 = 15b + 7$
n 3(mod 7)
$\Rightarrow 15b + 7 \equiv 3 \pmod 7 \Rightarrow 5b \equiv 1 \pmod 7$
$\Rightarrow b = 7c + 3$, where c is any positive number

$n = 15(7c + 3) + 7$
$\Rightarrow n = 105c + 52$
Since there are between 100 and 200 soldiers, n = 157

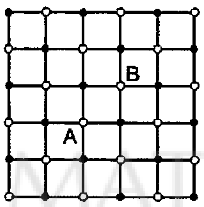
9) Let's assume according to the original plan, the working time for each day is x hours and the work done in unit time (an hour) is y. Then the work done each day is xy.
The work done each day after changing the working plan

$$= \left(1 - \frac{1}{4}\right)x \times \left(1 - \frac{1}{9}\right)y = \frac{5}{6}xy$$

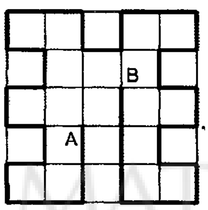
Let the number of days needed to finish the work according to the orignal plan be a.
Then number of days required to finish $\frac{2}{3}$ of the work $= \frac{2}{3}a$
Number of days required to finish the rest of the work $= \frac{1}{3} \times \frac{6}{5}a = \frac{2}{5}a$
Given,
$\frac{2}{3}a + \frac{2}{5}a = 32$
$\Rightarrow \frac{16}{15}a = 32$
$\Rightarrow a = 30$



10) There are 36 intersections in his home town, every time Ali passes a street; he comes to a new intersection. Hence he cannot go through more than 35 streets.




As illustrated in the diagram above, every time, he transverses a street, Ali comes to an intersection of different colour. Since A and B are of same colors, it follows that any route will have even number of streets. So, 35 streets are not possible either.
One possible solution with 34 streets is shown below:




11) $\frac{1}{7} = 0.\dot{1}4285\dot{7}$
$\frac{2}{7} = 0.\dot{2}8571\dot{4}$
$\frac{3}{7} = 0.\dot{4}2857\dot{1}$
$\frac{4}{7} = 0.\dot{5}7142\dot{8}$
$\frac{5}{7} = 0.\dot{7}1428\dot{5}$
$\frac{6}{7} = 0.\dot{8}5714\dot{2}$

Notice that the decimal representation of fractions of the form where a is less than 7 are repeating, non-terminating decimal with 6 digits 1, 4, 2, 8, 5, 7.
$1+4+2+8+5+7=27$
$9062 \equiv 17 \pmod{27}$ and $2014 \equiv 4 \pmod 6$
Since $5+7+1+4=17$, a = 4



12) Let $a_n$ denote a code of length 'n' such that no two 1s are together.
Note that $a_1=2$ and $a_2=3$.
For $n>2$, $a_n=a_{n-1}+a_{n-2}$, where $a_{n-1}$ is the no. that starts with 0 and $a_{n-2}$ is the no that starts with 01.
$a_3=5, a_4=8, a_5=13, a_6=21, a_7=34, a_8=55, a_9=89, a_{10}=144$



13) First observe that 74 divides 9990.
When 2000 is divided by 3, quotient = 666 and r = 2.
The required remainder = remainder when 99 is divided by 74 which is 25.

14) $\frac{1}{67}+\frac{1}{67+134}+\frac{1}{67+134+201}+...+\frac{1}{67+134+201+...+2010}$
$=\frac{1}{67}\left(\frac{1}{1}+\frac{1}{1+2}+\frac{1}{1+2+3}+...+\frac{1}{1+2+3+...+30}\right)$
$=\frac{2}{67}\left((1-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)+...+\left(\frac{1}{30}-\frac{1}{31}\right))$
$=\frac{2}{67}\left(1-\frac{1}{31}\right)$
$=\frac{60}{2077}$



15) We express the areas of triangles AFE, CDE and BDF as fractions of the area of triangle ABC.
Area of $\triangle$ AEF $=\frac{1}{5}\times\frac{5}{6}\times$ Area of $\triangle$ ABC $=\frac{1}{6}\times 150$
Area of $\triangle$ AEF $=\frac{1}{5}\times\frac{5}{6}\times$ Area of $\triangle$ ABC $=\frac{1}{6}\times 150$
Area of $\triangle$ BDF $=\frac{3}{4}\times\frac{1}{6}\times$ Area of $\triangle$ ABC $=\frac{1}{8}\times 150$
Area of $\triangle$ BDF $=\frac{4}{5}\times\frac{1}{4}\times$ Area of $\triangle$ ABC $=\frac{1}{5}\times 150$

Area of triangle DEF $=\left(1-\frac{1}{6}-\frac{1}{5}-\frac{1}{8}\right)\times 150=\frac{61}{120}\times 150=76\frac{1}{4}$ cm$^2$