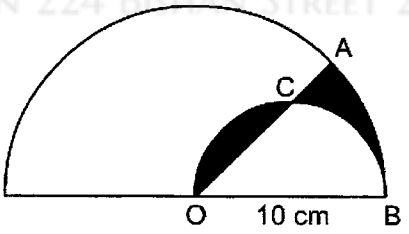
Mathlete Training Centre
Round 2 RIPMWC open

2010 RIPMWC open round 2

1.

In the figure, O is the centre of the bigger semicircle with radius 10 cm, OB is the diameter of the smaller semicircle and C is the midpoint of arc OB and it lies on the line segment OA. Taking $\pi = \frac{22}{7}$, find the area of the shaded region in cm^2.



2.

In a country called Binary, a mathematician comes up with a system of coins which can represent any integral amount from 1 to 200 cents without using any type of coins twice. What is the smallest number of types of coins in the system?

3.

Find the value of

$$\frac{2010.201 + 20102010 + 201020102010 + 2010201020102010}{2012.2012 + 20122012 + 201220122012 + 2012201220122012}$$

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4.

I have a number in my mind. After doubling it, I subtract 5 from it before dividing the answer by 3. Then, I multiply the result by itself and subtract 192 from it. Finally, I obtain the largest 2-digit prime number. What is the original number that I had in mind?

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5.

The clock on the wall shows 11 am now, find the time in minutes that would elapse when minute hand and hour hand first forms a right angle.

6.

What is the sum of all 4-digit numbers in which digits 1, 2, 4, 5 appear exactly once?

7.

John and Terry share a plot of land. The ratio of the area of John's portion to the area of Terry's portion is 3 : 2. They each grow barley and corn on their piece of land. The entire plot of land is covered by barley and corn in the ratio of 7 : 3. On John's portion of the land, the ratio of barley to corn is 4 : 1. What is the ratio of barley to corn on Terry's portion?

8.

When a commander asks his soldiers to march in rows of 3, one soldier is left out; when he asks them to march in rows of 5, 2 soldiers are left out; when he asks them to march in rows of 7, 3 soldiers are left out. If there are between 100 and 200 soldiers, how many soldiers are there?

9.

In completing a certain project, workers worked according to original plan for the first $\frac{2}{3}$ of the work to be done. Some changes were made to the work-plan. As a result, the working time each day was reduced by 25%, but the work done in a unit time was increased by $11\frac{1}{9}\%$. It took 32 days to finish the work in the end. If they complete the work according to the original plan, how many days are needed to finish the work?

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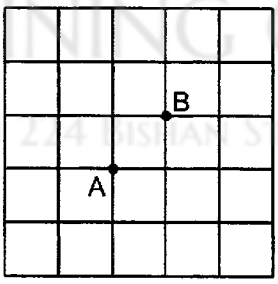
10.

On the eve of his departure to work overseas, in his moment of nostalgia, Ali decided to walk on the streets of his home town from his home (point A on the map below) to the train station (point B) using the longest way possible but never passing through the same point twice. (He can only move on the grid.) Assuming any interval of length 1 to be a street, how many streets can Ali traverse?



11.

When the fraction $\frac{a}{7}$, where a is a positive integer less than 7, is expressed as a decimal, the sum of the 2014 digits after the decimal is 9062. Find a.

12.

A code is to be formed by using only digits 0 and 1. The length of the code is the number of digits in the code. Find the number of codes of length 10 such that no two 1s can be together (For example, 0101010010).

13.

What is the remainder when $\underbrace{999...999}_{\text{2000 9s}}$ is divided by 74?

14.

Find the sum of

$$\frac{1}{67}+\frac{1}{67+134}+\frac{1}{67+134+201}+...+\frac{1}{67+134+201+...+2010}.$$

15.

In the ΔABC shown below, points D, E and F are on sides BC, CA and AB respectively such that BF : AB = 1 : 6, AE : AC = 1 : 5 and CD : CB = 1 : 4. Given that the area of triangle ABC is 150 cm^2, find the area of triangle DEF in cm^2.