Mathlete Training Centre
Round 1 RIPMWC Open

2009 RIPMWC Open round 2 Answers

1) Difference in areas

$= \frac{\pi(2)^2}{4} - \left(2^2 - \frac{\pi(2)^2}{4}\right)$

$= 2 \times \frac{22}{7} - 4$

$= \frac{16}{7}$

$= 2\frac{2}{7}$ cm^2

2) Rate of completion by $A + B = \frac{5}{6} \div 6 = \frac{5}{36}$ $36 \div 5 = 7.2$

After A and B worked for $7 \times 2 = 14$ days, A left with $1 - 7 \times \frac{5}{36} = \frac{1}{36}$ of the project to complete.

Since A completes $\frac{1}{3} = B$ completes $\frac{1}{2}$

A's rate of completion is $\frac{2}{3}$ of B.

A's rate of completion $= \frac{5}{36} \times \frac{2}{5} = \frac{1}{18}$

He will take $\frac{1}{36} \div \frac{1}{18} = \frac{1}{2}$ days to complete.

Hence the total number of days to complete $= 14\frac{1}{2}$ days

3) By selecting any two vertical lines and two horizontal lines, a unique rectangle is determined.

Hence the total number of ways would be $\frac{6 \times 5}{2} \times \frac{7 \times 6}{2} = 315$

4) Let the original time be x h, since the speed decreases by 10%, the time will decrease by $\frac{10}{9}$ which is $\frac{10x}{9}$ h

$\frac{10x}{9} - x = 1$

$x = 9$

Let the original speed be v km/h.

$\frac{180}{v} + \frac{9v - 180}{1.2v} = 9 - 1$

$\frac{180}{v} + 7.5 - \frac{150}{v} = 8$

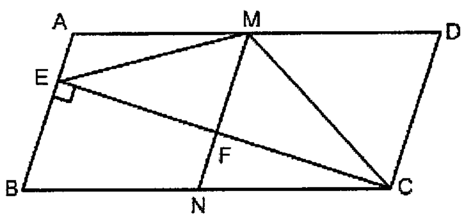
$\frac{30}{v} = 0.5$

$v = \frac{30}{0.5} = 60$
Thus the distance between A and B is $60 \times 9 = 540$ km.

5) $\frac{12}{4\times 5} + \frac{12}{10\times 8} + \frac{12}{16\times 11} + ... + \frac{12}{4012\times 2009}$
$= \frac{6}{2\times 5} + \frac{6}{5\times 8} + \frac{6}{8\times 11} + ... + \frac{6}{2006\times 2009}$
$= \frac{6}{3}\left[\left(\frac{1}{2} - \frac{1}{5}\right) + \left(\frac{1}{5} - \frac{1}{8}\right) + \left(\frac{1}{8} - \frac{1}{11}\right) + ... + \left(\frac{1}{2006} - \frac{1}{2009}\right)\right]$
$= 2\left[\frac{1}{2} - \frac{1}{2009}\right]$
$= \frac{2007}{2009}$

6) In each replacement, the highest common factor will not be changed.
Hence the last two numbers must be the highest common factor.
$2009 = 7^2 \times 41$
$9002 = 2 \times 7 \times 643$
Hence the HCF IS 7.
So the last two numbers are $(7, 7)$.

7) Choose a point N on BC such that $MN \parallel AB$.
Let MN intersect CE at F and join M to C.
As M is the midpoint of AD and $MN \parallel AB$, $BN = NC$
Since $NF \parallel BE, CF : FE = CN : NB = 1 : 1, CF = EF$.
Also, $CFN = CEB = 90°$
Since MF is the perpendicular bisector of CE,
$CMF = EMF$
But $EMF = AEM = 90° - 35° = 55°$,
$CMF = 55°$
Since $MN = CD = MD$, so $CDMN$ is a rhombus.
Hence $CMD = CMF = 55°$
Therefore $DME = 3 \times 55° = 165°$



8) Let the fixed amount of sugar solution transferred be x litres.

$$\frac{50 \times 10\% - 10\%x + 20\%x}{50} = \frac{30 \times 20\% - 20\%x + 10\%x}{30}$$

$$\frac{5 + 0.1x}{50} = \frac{6 - 0.1x}{30}$$

$150 + 3x = 300 - 5x$

$8x = 150$

$x = 18.75$

Hence the fixed amount transferred is 18.75 litres.

9) $$\frac{2 \times 4 \times 6 + 6 \times 12 \times 18 + 10 \times 20 \times 30 + 14 \times 28 \times 42 + 18 \times 36 \times 54}{1 \times 3 \times 5 + 2 \times 6 \times 10 + 4 \times 12 \times 20 + 8 \times 24 \times 40}$$

$$= \frac{2 \times 4 \times 6 + (1 + 3^3 + 5^3 + 7^3 + 9^3)}{1 \times 3 \times 5 + (1 + 2^3 + 4^3 + 8^3)}$$

$$= \frac{2 \times 4 \times 6 \times 1225}{1 \times 3 \times 5 \times 585}$$

$$= \frac{2 \times 4 \times 2 \times 49}{117}$$

$$= \frac{784}{117}$$

$$= 6\frac{82}{117}$$

10) Every team has 9 matches to play.

The total number of matches is $9 \times 10 \div 2 = 45$

No matter what the result is, a sum of 2 points will be rewarded to both teams.

Thus the total number of points is 90 points.

Let their total number of points be a_n where n = 1, 2, ..., 10

Then $a_1 > a_2 > a_3 > ... > a_9 > a_{10}$

Since the top two teams never lost, the top team must have won at most 8 matches and draw 1 match.

$a_1 \leq 8 \times 2 + 1$, i.e. $a_1 \leq 17$

Therefore $a_2 \leq 16$, $a_3 = a_1 + a_2 - 20$ which means that $a_3 \leq 13$

Next, let us eliminate the condition of $a_3 < 13$

If $a_3 < 13$, then $a_3 \leq 12, a_4 \leq 11, a_5 \leq 10, a_6 \leq 9$ and $a_7 + a_8 + a_9 + a_{10} = a_4 \leq 11$

The total number of points of all 10 teams $\leq 17 + 16 + 12 + 11 + 10 + 9 + 11$

i.e total number of points of all 10 teams $\leq 86 < 90$, this is impossible

Thus $a_3 = 13$

Hence total number of points obtained by the third ranking team is 13 points.

11) Suppose that Paul has removed some balls from the bag, and the remaining balls do not satisfy the required condition. What is the maximum number of balls that can remain?

In order to not satisfy the required condition, either there are not 4 balls of any colour(so the maximum number is 9 balls, ie, 3 of each colour) or there are at least 4 balls of one colour, but there are not 3 of either of the other colours.

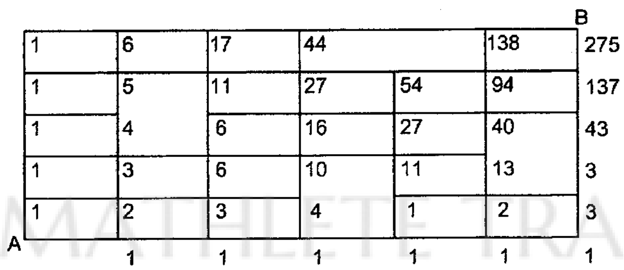
In this second case, we could have 2 balls of each of two colours, and as many as possible of the third colour. The maximum number of balls of any colour that can be in the bag is 8 (the

number of yellow balls which Paul starts).
So the maximum number of balls still in the bag in this case is 12.

Therefore, if Paul removes 8 or more balls, then the remaining balls might not satisfy the required condition.

12) Let the answer be A.
$\frac{0.2468}{0.92} < A < \frac{0.247}{0.919}$
$0.2682... < A < 0.2687...$
The answer is 0.2686. i.e. 0.269

13) Answer as follows:



14) Count upright facing triangles, then downward facing ones.
Upward $\triangle$s $= 1 + 3 + 6 + 10 + 15 + 21 + 28 + 36 = 120$
This can be done by considering that there are 36 triangles of size 1, 28 triangles of size 2 and so on.

Downward $\triangle$s $= 1 + 6 + 15 + 28 = 50$
This is a bit trickier. There are 28 size 1 triangles, 15 size 2 triangles, 6 size 3 triangles and 1 size 4 triangle. Notice that this is the sum of alternate triangle numbers.
Note that it starts with the 7^{th} triangle number.
Hence total number of triangles = 120 + 50 = 170

15) Let h be the height of trapezoid $ABCD$.
Since $AB = 2$ and AX is parallel to BC, then $XC = 2$.
Since $AB = 2$ and BY is parallel to AD, then $DY = 2$
Since $CD = 5$, $XC = 2$ and $DY = 2$, then $YX = 1$
Then the total area $= \frac{1}{2}(AB + CD)h = \frac{7}{2}h$

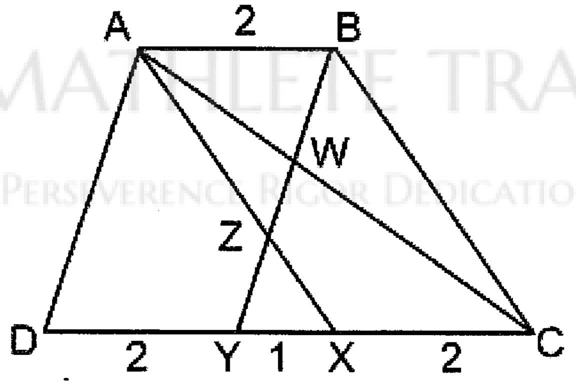
Now we want to determine the area of $\triangle AZW$,
so we will determine the areas of $\triangle AZB$ and $\triangle AWB$ and subtract them.

First we calculate the area of $\triangle AZB$. Since $AB \parallel CD$, $ZAB = ZXY$ and $ZBA = ZYX$, so $\triangle AZB$ is similar to $\triangle XZY$. Since the ratio of AB to XY is 2 to 1, then the ratio of the heights of these 2 $\triangle$s will also be 2 to 1, since they are similar.

But the sum of their height must be the height of the trapezoid, h, so the height of $\triangle AZB$ is $\frac{2}{3}$h.
Therefore the area of $\triangle AZB$ is $\frac{1}{2}(2)(\frac{2}{3}h) = \frac{2}{3}h$.

Next we calculate the area of $\triangle AWB$. Since $AB \parallel CS$, $WAB = WCY$ and $WBA = WYC$, so $\triangle AWB$ is similar to $\triangle CWY$. Since the ratio of AB to CY is 2 to 3, then the ratio of the heights of these 2 $\triangle$s will also be 2 to 3, since they are similar. But the sum of their heights must be the height of the trapezoid,h, so the height of $\triangle AWB$ is $\frac{2}{5}h$.

Therefore, the area of $\triangle AZB$ is $\frac{1}{2}(2)(\frac{2}{5}h) = \frac{2}{5}h$.
Therefore, the area of $\triangle AZW$ is the difference between the areas of $\triangle AZB$ and $\triangle AWB$, or $\frac{2}{3}h - \frac{2}{5}h = \frac{4}{15}h$
Thus, the ratio of the area of $\triangle AZW$ to the area of the whole trapezoid is
$\frac{4}{15}h : \frac{7}{2}h = 4(2) : 7(15) = 8 : 105$



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