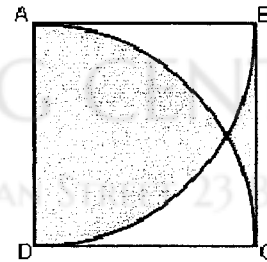Mathlete Training Centre
Round 2   RIPMWC open

2009 RIPMWC open round 2

1.

In the given figure, each side of the square ABCD is 2 cm.

Calculate the difference of the areas of two shaded regions in cm².
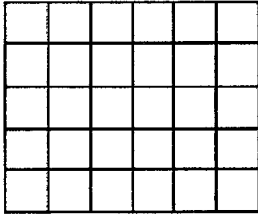
(Take $\pi = \dfrac{22}{7}$)

2.

If workers A and B work together for 6 days, they can complete $\dfrac{5}{6}$ of a given engineering project.

The time taken for A working alone to complete $\dfrac{1}{3}$ of the project is the same as that for B working

alone to complete $\dfrac{1}{2}$ of the project. If A and B take turns to work on the project every day,

i.e. A, B, A, B, ..., how many days will they take to complete the project?

3.

How many rectangles can be formed in the following diagram?

4.

Victor drives a car from A to B. If he decreases his speed by 10%, he will arrive at B one hour later than the original time. If he drives at his original speed for the first 180 km and then increases his speed by 20% for the remaining journey, he will arrive at B one hour earlier. Find the distance between A and B in km.

5.

Find the value of the sum

$$\frac{12}{4 \times 5} + \frac{12}{10 \times 8} + \frac{12}{16 \times 11} + \cdots + \frac{12}{4012 \times 2009}$$

6.

Starting from a pair of natural numbers (a, b), an operation which replaces the larger number by the difference of the two numbers is applied. This operation is repeated until the resulting two numbers are the same. For example, if we start with (6, 8), then 8 will be replaced by 8 − 6, i.e. 2. So the number becomes (6, 2). The operation is repeated and the resulting numbers are shown below:

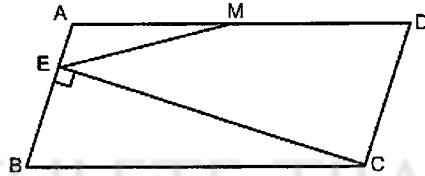$$(6, 8) \longrightarrow (6, 2) \longrightarrow (4, 2) \longrightarrow (2, 2)$$

Apply this operation to (2009, 9002). What are the last two numbers when the operation stops?

7.

In a parallelogram ABCD, AD = 2AB. Point M is the midpoint of segment AD.
E is a point on AB and CE is perpendicular to AB. If ÐCEM = 35°, find ÐDME in degrees.

8.

There are two containers, A and B. Container A has 50 litres of water with 10% concentration
of sugar. Container B has 30 litres of water with 20% concentration of sugar.
Same volume of sugar solution is removed from each container and put into the other container.
After the transfer, the concentration of sugar in both containers become the same.
Find the volume of sugar solution (in litres) transferred.

9.

Find the value of the sum

$$\frac{2 \times 4 \times 6 + 6 \times 12 \times 18 + 10 \times 20 \times 30 + 14 \times 28 \times 42 + 18 \times 36 \times 54}{1 \times 3 \times 5 + 2 \times 6 \times 10 + 4 \times 12 \times 20 + 8 \times 24 \times 40}$$

10.

In a soccer competition, all ten teams played against one another. The number of points awarded for a win, draw and lose are 2, 1 and 0 points respectively. At the end of the competition, it is known that the total number of points obtained by the ten teams are all different; the top two teams never lost a single match and the sum of their total number of points is 20 points more than the total number of points obtained by the third ranking team; the total number of points of the fourth ranking team is the sum of the total number of points of the last 4 teams. What is the total number of points obtained by the third ranking team?

11.

A bag contains eight yellow marbles, seven red marbles, and five black marbles.
Without looking in the bag, Paul removes N marbles all at once. If he is to be sure that,
no matter which combination of N marbles he removes, there are at least four marbles of
one colour and at least three marbles of another colour left in the bag, what is the
maximum possible value of N?

12.

Divide 2469.2469 by 9192.9192, what is the answer correct to 3 decimal places?

13.

How many shortest ways are there to get from point A to point B?
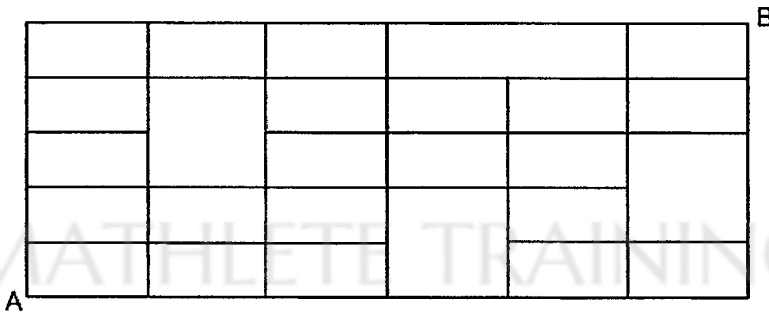
14.

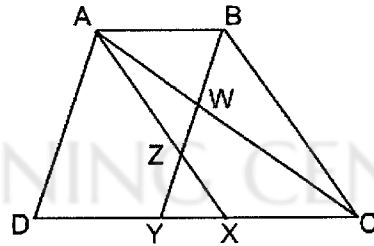A series of patterns are made by drawing equilateral triangles to form a pyramid as such:

Figure 1   Figure 2   Figure 3
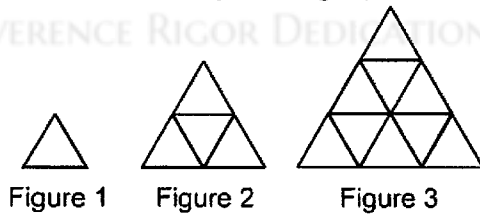
The table below shows the number of equilateral triangles for Figures 1, 2 and 3.

Figure number	1	2	3
Equilateral triangles	1	5	13

Find the total number of equilateral triangles in Figure 8.

15.

In the diagram, ABCD is a trapezium with AB parallel to CD and with AB = 2 and CD = 5.
Also, AX is parallel to BC and BY is parallel to AD. If AX and BY intersect at Z, and AC and BY
intersect at W, find the ratio of the area of $\triangle$AZW to the area of trapezium ABCD.