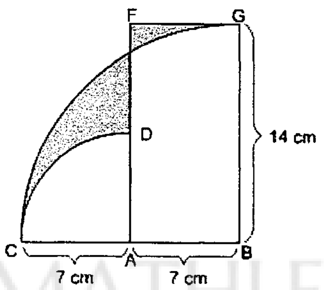
Mathlete Training Centre
Round 2 RIPMWC Open

**2008 RIPMWC Open Round 2 Answers**

1) Number of members in group A = 36n + 11, where n is an integer
Number of members in group B = 36m + (36 −11) = 36m + 25, where m is an integer
Number of pictures that the members would take
$= (36n + 11) \times (36m + 25)$
$= 36n \times (36m + 25) + 11 \times 36m + 11 \times 25$
$= 36n \times (36m + 25) + 11 \times 36m + 7 \times 36 + 23$
$= 36p + 23$, for some integer p
So the number of remaining pictures that can be taken from the last roll = 36 − 23 = 13

2) Number of ways for 2 pupils to get correct test scores $= \frac{5 \times 4}{2} = 10$
Number of ways for 3 pupils to get wrong test scores = 2
Total number of ways in which exactly 3 among the 5 students get the wrong scores
$=10 \times 2$
$=20$

3) Refer to diagram:




Difference in areas of the shaded region
= area of quadrant BCG − area of quadrant ACD − area of rectangle ABGF
$=\frac{\pi(14)^2}{4} - \frac{\pi(7)^2}{4} - 7 \times 14$
$=49 \times \frac{22}{7} - \frac{49}{4} \times \frac{22}{7} - 98$
$=154 - 38\frac{1}{2} - 98$
$=17\frac{1}{2}$
$=17.5$

4) Let the cost price for a batch of goods be \$x.
Then the profit on 70% of the goods = $50\% \times 70\% \times \$x = \$0.35x$
Suppose Mr Khoo sells the remaining goods for \$y for a batch.
Then profit on 30% of the goods = $(y-1) \times x \times 30\% = \$0.3\ (y-1)x$
Total profit = $\$0.3\ (y-1)x + \$0.35x = 82\% \times 50\% \times x$
$y = 1.2$
So, percentage discount $= \dfrac{1.2}{1.5} \times 100 = 80\%$



5) $\left(\frac{1}{2}+\frac{1}{3}+...+\frac{1}{60}\right)+\left(\frac{2}{3}+\frac{2}{4}+...+\frac{2}{60}\right)+\left(\frac{3}{4}+\frac{3}{5}+...+\frac{3}{60}\right)+...+\left(\frac{58}{59}+\frac{50}{60}\right)+\left(\frac{59}{60}\right)$
$=\left(\frac{1}{2}\right)+\left(\frac{1}{3}+\frac{2}{3}\right)+\left(\frac{1}{4}+\frac{2}{4}+\frac{3}{4}\right)+\left(\frac{1}{5}+\frac{2}{5}+\frac{3}{5}+\frac{4}{5}\right)+...+\left(\frac{1}{60}+\frac{2}{60}+\frac{3}{60}+...\frac{59}{60}\right)$
$=\frac{1}{2}+\frac{1}{3}\times\frac{(3-1)\times(1+2)}{2}+\frac{1}{4}\times\frac{(4-1)\times(1+3)}{2}+\frac{1}{5}\times\frac{(5-1)\times(1+4)}{2}+...+$
$\frac{1}{60}\times\frac{(60-1)\times(1+59)}{2}$
$=\frac{1}{2}+\frac{2}{2}+\frac{3}{2}+\frac{4}{2}+...+\frac{59}{2}$
$=\frac{1}{2}(1+2+...+59)$
$=\frac{1}{2}\times\frac{59\times 60}{2}$
$=885$



6) Case 1: The three digit number has no zero
No. of three digit numbers having no zeros = $9 \times 1 \times 8 \times 3 = 216$
Case 2: The three digit number has one zero
No. of three digit numbers having one zero = $9 \times 1 \times 1 \times 2 = 18$
Case 3: The three digit number has two zeroes
No. of three digit numbers having two zeros = $9 \times 1 \times 1 = 9$
Total number of possible 3-digit numbers with two identical digits = $216 + 18 + 9 = 243$

7) No. of blue marbles removed $= \frac{5}{5+8} \times 91 = 35$
No. of green marbles removed $= 91 - 35 = 56$
Let x and y be the number of remaining blue and green marbles.
$x = \frac{4}{3}y$
$x + 35 = \frac{3}{4}(y + 56)$
$\frac{4}{3}y + 35 = \frac{3}{4}y + 42$
$\frac{7}{12}y = 7$
$y = 12$
$x = 16$
Therefore the total number of marbles $= 91 + 12 + 16 = 119$



8) $\frac{1^2+2^2}{1\times 2} + \frac{2^2+3^2}{2\times 3} + \frac{3^2+4^2}{3\times 4} + \ldots + \frac{2007^2+2008^2}{2007\times 2008}$
$=\left(\frac{1}{2}+\frac{2}{1}\right) + \left(\frac{2}{3}+\frac{3}{2}\right) + \left(\frac{3}{4}+\frac{4}{3}\right) + \ldots + \left(\frac{2007}{2008}+\frac{2008}{2007}\right)$
$=2 + \left(\frac{1}{2}+\frac{3}{2} + \left(\frac{2}{3}+\frac{4}{3}\right) + \left(\frac{3}{4}+\frac{5}{4}\right) + \ldots + \left(\frac{2006}{2007}+\frac{2008}{2007}\right) + \frac{2007}{2008}\right.$
$==2 \times 2007 + \frac{2007}{2008}$
$=4014\frac{2007}{2008}$
$\approx 4015$



9) The number of telephone lines can be written as
$1, 1+2, 1+2+3, 1+2+3+4, \ldots$
Hence each term is a sum of terms
i.e for 5 teachers, there are $1+2+3+4 = 10$ lines
Hence if there are n teachers, there are $1+2+3+\ldots+(n-1) = \frac{1}{2}n(n-1)$ lines
$\frac{1}{2}n(n-1) = 190$
$n(n-1) = 380$
By observation, $n = 20(as 19 \times 20 = 380)$
Hence there are 20 teachers.



10) Suppose that one ox eats x unit grass in a week and the grass grows y unit a week.
Let there be z unit of grass on the farm originally.
If it takes t weeks for 21 oxen to finish, then
$z + 6y = 27(x)(6) \Rightarrow z + 6y = 162x$ —(1)
$z + 9y = 23(x)(9) \Rightarrow z + 9y = 207x$ —(2)
$z + ty = 21(x)(t) \Rightarrow z + ty = 21xt$ —(3)

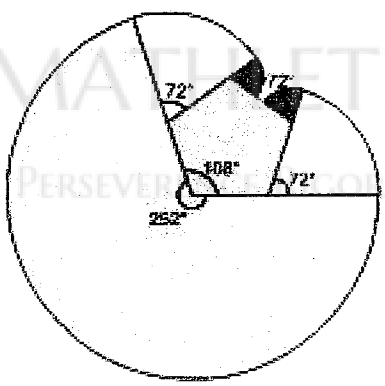
(2) - (1): 3y = 45x ⇒ y = 15x —(4)
Sub (4) into (1): z + 90x = 162x ⇒ z = 72x —(5)
Sub (4) and (5) into (3): 72x + 15xt = 21xt
$t = \frac{72}{6} = 12$
It takes 12 weeks for 21 oxen to finish grass on the farm.

11) Let $a_n$ be the no of ways of reaching step n.
$a_1 = 1$ [(1), i.e. one 1-step]
$a_2 = 2$ [(1,1), (2), two 1-step or one 2-step]
$a_3 = 4$ [(1,1,1), (2,1), (1,2), (3)]
$a_4 = 7$ [(1,1,1,1), (2,1,1), (1,2,1), (1,1,2), (2,2),(3,1),(1,3)]
$= 1 + 2 + 4 = a_1 + a_2 + a_3$
$a_5 = a_2 + a_3 + a_4$ [(1,1,1,1,1), (2,1,1,1) ×4, (2,2,1) × 3, (3,1,1) × 3, (3,2) × 2]
$= 2 + 4 + 7 = 13$
$a_6 = a_3 + a_4 + a_5 = 4 + 7 + 13 = 24$
$a_7 = a_4 + a_5 + a_6 = 7 + 13 + 24 = 44$
$a_8 = a_5 + a_6 + a_7 = 13 + 24 + 44 = 81$
$a_9 = a_6 + a_7 + a_8 = 24 + 44 + 81 = 149$



12) $4 \times 8 \times 12 \times 16 \times ... \times 2004 \times 2008$
$= 4^{502}(1 \times 2 \times 3 \times ... \times 501 \times 502)$
No. of zeros $= \frac{502}{5} + \frac{502}{25} + \frac{502}{125} = 100 + 20 + 4 = 124$

13) The diagram shows the region that the goat can reach.
Interior angle of a pentagon $= 180° - \frac{360°}{5} = 108°$

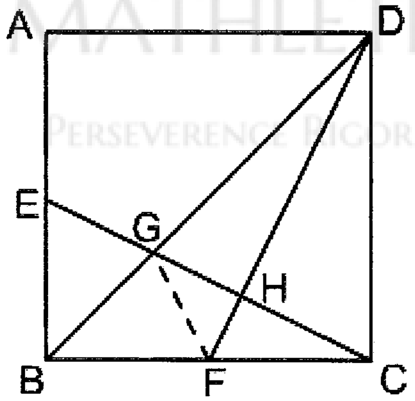



Required area $= \frac{252°}{360°} \times \pi \times (7)^2 + \frac{72°}{360°} \times \pi \times (4)^2 \times 2 + \frac{72°}{360°} \times \pi \times (1)^2 \times 2$
$=34\frac{3}{10}\pi + 6\frac{2}{5}\pi + \frac{2}{5}\pi$
$=41\frac{1}{10}\pi$
$=41.1\pi$ cm$^2$

Hence the area is $41.1\,\pi$ cm$^2$

14) If she wants the 3 smallest lengths that cannot form a triangle,
Esther should start with lengths 1,1 & 2 (first 3 Fibonacci numbers)
By adding the last 2 numbers in the sequence to form the third, she would get the sequence $1, 1, 2, 3, 5, 8, 13, 21, 34$
Notice that, by triangle inequality, if we take any 3 lengths in this sequence, we can never form a triangle.
Hence the shortest possible length of the longest stick is 34.

15) Area of $\triangle$ BEC = Area of $\triangle$ BFD = Area of $\triangle$ CDF = $\frac{1}{4}\times$ Area of ABCD = $\frac{1}{4}\times 120 = 30$ cm$^2$.
Join FG
Area of $\triangle$ BFG = Area of $\triangle$ CFG = Area of $\triangle$ BEG = $\frac{1}{3}\times$ Area of $\triangle$ BEC = $\frac{1}{3}\times 30 = 10$ cm$^2$
$\therefore$ Area of $\triangle$ DFG = Area of $\triangle$ BFD - Area of $\triangle$ BFG = 30 - 10 = 20 cm$^2$



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