Mathlete Training Centre
SMOPS 2023

Round 1

1. A wonderful number is divisible by 49 and consists of only digits 4 and 9. 49 is the smallest wonderful number. What is the 2^{nd} smallest wonderful number?

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2. Find the sum of the digits of 10^{23} - 20.

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3. An integer A is written as the product of all its individual prime factors. What is the largest value of A whose individual prime factors add up to 14? (For example, 12 = 2*2*3, the sum of its individual prime factors is 2+2+3=7.)

4. Given that $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=1$ and a,b,c are positive integers, how many different values of a+b+c are there?

5. What is the smallest possible number of dates for the second Tuesday after the second Friday in a month?

6. There is a glass of water. In the first step, $\frac{1}{2}$ of water was poured out; in the second step, $\frac{1}{3}$ of the remaining water was poured out; in the third step, $\frac{1}{4}$ of the remaining water was poured; in the fourth step, $\frac{1}{5}$ of the remaining water was poured out... After which step will there be $\frac{1}{10}$ of the original amount of water left in the glass?

7. How many four-digit number $\overline{ABCD}$ are there such that A+B=C+D and A+C=B+D?

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8. Mr Wong travels from A to B at a speed of 60km/h. He then travels from B to C without stopping at B at 45km/h. Given that AB:BC = 2:1, find Mr Wong's average speed throughout the journey in km/h.

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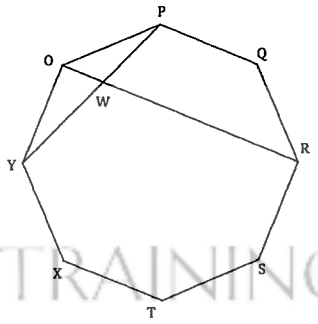
9. If PQRSTXYO is a regular 8-sided polygon and $\angle PWR = x$, find the value of 4x.



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10. There are 27 1×1×1 cubes such that their six sides are labelled with 1 to 6 according to the following rule: the opposite side of 1 is 2, the opposite side of 3 is 4 and the opposite side of 5 is 6. If these 27 1×1×1 cubes were stacked to form a 3×3×3 cube and S is the sum of the numbers on the outer surface of the 3×3×3 cube, what is the maximum value of S?

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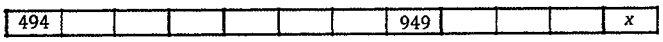
11. In the following table, the sum of any 3 adjacent cells is 2023, find x.



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12. An n-sided polygon has 5 times as many diagonals as sides. Find n. (For example, a square has 4 sides and 2 diagonals)

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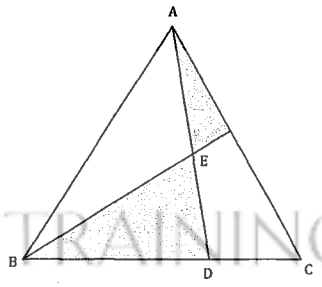
13. At a carnival, an adult ticket costs \$4 more than a child ticket. One day, they sold 100 more child tickets and made \$1475. How much would they have made that day if the prices of the child and adult tickets were swapped?

14. From 1 to 1000, how many numbers have 9 divisors?

15. All square numbers from 1 to 10000 are written in 1 line: 1491625...10000. How many digits are there?

16. In Oldcastle University, there are 900 students. Each student learns 6 subjects. Each teacher teaches 5 subjects. If every class consists of 30 students and 1 teacher, how many teachers are there?

17. As shown in the figure below, given that the area of $\triangle ABC = 64cm^2$, 2BD = 3DC and AE = ED, find the area of the shaded region.



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18. If the lowest common multiple of a,b and c is 2023 times the highest common factor, what is the lowest sum of a+b+c?

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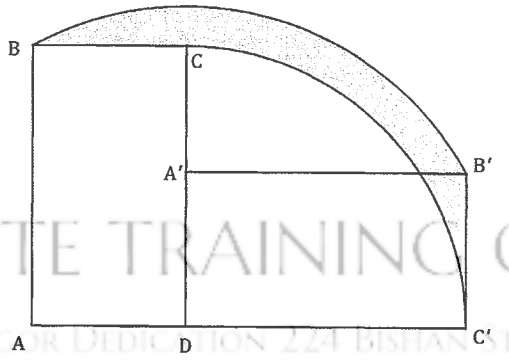
19. Tom writes 11 consecutive numbers on a piece of paper. If he erases one of the numbers, the sum of the remaining numbers is 2023. Find the number that he erased.

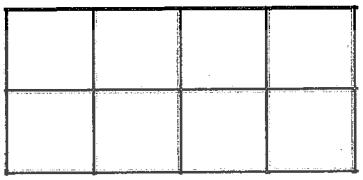
20. Given the sequence $\frac{1}{2},\frac{1}{3},\frac{2}{3},\frac{1}{4},\frac{2}{4},\frac{3}{4},\frac{1}{5},\ldots$. If the 2023^{rd} number of this pattern is $\frac{a}{b}$, what is a+b?

21. 2023 is divisible by 7 and 2024 is divisible by 8. When will the year after 2023 be divisible by 7, and the next year be divisible by 8?

22. A rectangle ABCD is rotated 90° clockwise around point D to position A'B'C'D. If AB = 41cm, AD = 14cm, find the shaded area.



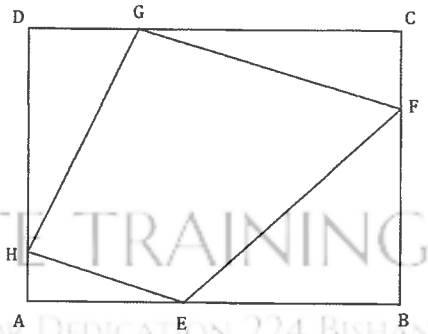
23. How many ways are there to colour to 2×4 grid below using 4 different colours such that each cell is coloured with only one colour and the colour of adjacent cells must be different.



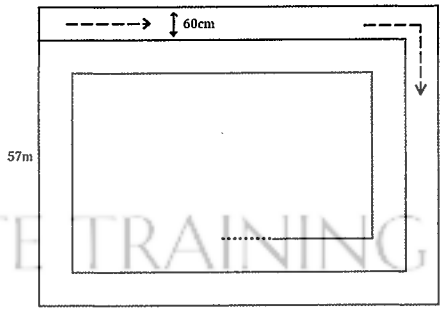
24. 4 identical red balls and 4 identical blue balls are arranged in a straight line. How many arrangements are there if every 3 balls must have 2 different colours?

25. How many 5-digit numbers are there that are divisible by 990 and all five digits must be distinct?

26. As shown in the figure below, given that AB = 60, BC = 40, H is 24cm lower than F, G is 10cm to the left of E, what is the area of the quadrilateral EGFH?



27. Adam is mowing a garden of 108m long and 57m wide. His grass mower is 60cm wide. At least how many right turns does he have to take to completely mow the garden?



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28. A 5-digit number $\overline{abcde}$ is called a "rollercoaster" number if it meets all the conditions:
a, b, c, d and e are distinct numbers from 1 to 9.
d is larger than its 2 neighbouring numbers.
b is larger than its 2 neighbouring numbers.
How many such "rollercoaster" numbers are there?

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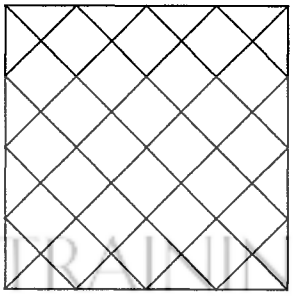
29. How many triangles are there in the following figure?



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30. In the following figure, how many rectangles (including squares) are there where the sum is a multiple of 7?

43	44	45	46	47	48	49
36	37	38	39	40	41	42
29	30	31	32	33	34	35
22	23	24	25	26	27	28
15	16	17	18	19	20	21
8	9	10	11	12	13	14
1	2	3	4	5	6	7

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