Mathlete Training Centre

**SMOPS 2022 Questions**

Questions

1. Jason drinks 70% of the water in a bottle and then refills 100 ml. Now the amount of water in the bottle is half of the initial amount. Find the initial volume (in ml) of water in the bottle.

2. There are 30 marbles in a bag, comprising 10 of each colour, red, yellow and green. Each red, yellow and green marble weighs 4 grams, 5 grams and 6 grams respectively. Eight marbles are now selected randomly from the bag, with a total mass of 39 grams. What is the maximum possible number of red marbles selected?

3. A list of numbers are defined as follows. The first number is 2000, the second number is 2022. From the third number onwards, every number is the average of the two preceding numbers. What is the integer part of the 15th number? (For example, the integer part of 2.65 is 2.)

4. The diagram shows a cube with side length 6 cm. If a rectangular tunnel with dimensions 2cm by 3cm is made in the middle of the cube, find the amount of increase in the total surface area of the resulting solid in $cm^2$.
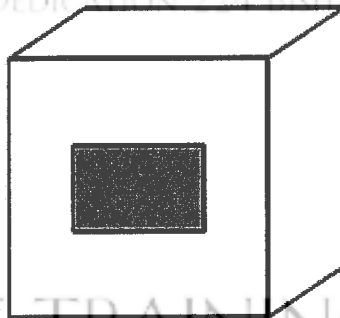
5. Three chess pieces, each of the colour red, black and white, are to be placed on a $6 \times 6$ chess board. If any two of the three pieces cannot be placed in the same row or the same column, how many ways are there to place the three chess pieces?

6. A logistic company is tasked to transport 89 tonnes of cargo. The capacity of lorry and caravan is 7 tonnes and 4 tonnes respectively. If each lorry consumes 14 litres of gasoline for the trip while each caravan only uses 9 litres, what is the least total gas consumption (in litres) to complete the task?

7. Alan and Betty started running towards each other at the same instant, from cities A and B respectively. The ratio of Alan's speed to Betty's is 3:2. Given that they meet at a point that is 18km away from the midpoint of AB, find the distance between A and B.

8. The diagram shows a parallelogram ABCD. E is the midpoint of AD. F is the midpoint of EC. If the area of the triangle BFD is 12 $cm^2$, find the area of the parallelogram ABCD in $cm^2$.
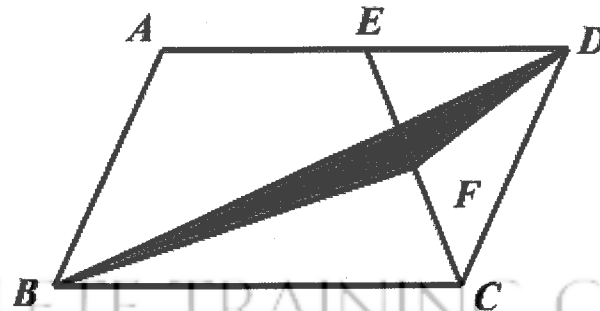
9. The sum of 49 distinct positive integers is 2022. If there are $n$ even numbers among the 49 integers, what is the least possible value of $n$?

10. A semi-circle with diameter AG is shown in the diagram below. The entire arc of the semicircle is divided into 6 equal parts by points B, C, D, E and F. DF and CG are straight lines. Given that the area of the semi-circle is $84cm^2$, find the area of the shaded region in $cm^2$.
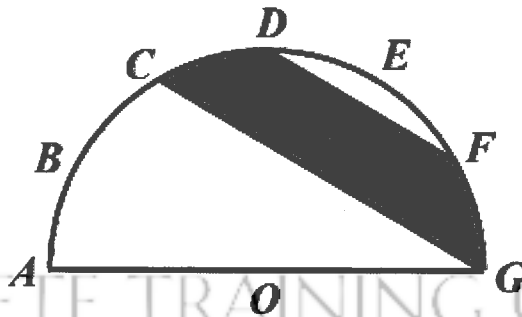
11. Given four prime numbers $a$, $b$, $c$ and $d$. If the product of $a \times b \times c \times d$ is the sum of 77 consecutive positive integers, find the smallest possible value of $a+b+c+d$.

12. The diagram shows a $5 \times 5$ square with 25 unit squares. Find the least number of unit squares to be shaded such that any $3 \times 3$ square in the diagram contains exactly four shaded unit squares.
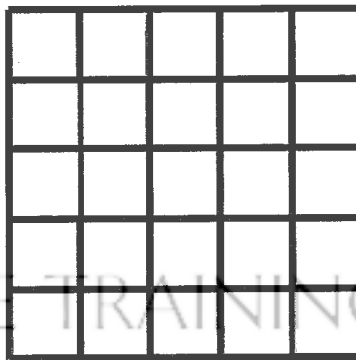
13. There are five cards A, K, Q, J, 10 and five envelopes labelled 'A', 'K', 'Q', 'J' and '10'. In the dark, Amy randomly inserts one card into every envelope. How many different ways are there such that every card is in the wrong envelope?

14. Find the smallest multiple of 35 that ends with '35' and has a sum of digits equal to 35. (If your answer is more than 5 digits, shade the first 5 digits. For example, if the answer is 1234535, please shade 12345.)

15. In the following number formation, if 2022 is the $n^{th}$ number in the $m^{th}$ row, find the value of m+n.

$$
\begin{array}{ccccccc}
 & & & 2 & & & \\
 & & 4 & 6 & 8 & & \\
 & 10 & 12 & 14 & 16 & 18 & \\
20 & 22 & 24 & 26 & 28 & 30 & 32 \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots
\end{array}
$$