Mathlete Training Centre
AMC 2022 Senior

1. The temperature in the mountains was 4°C but dropped overnight by 7°C. What was the temperature in the morning?

2. The rectangle shown has area 135. What is the value of h in the diagram?
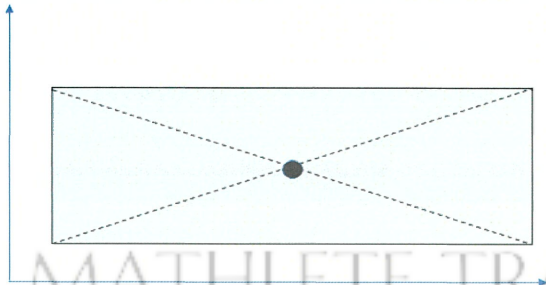
Area = 135 h

15

3. What is the value of $5^1 + 4^2 + 3^3 + 2^4 + 1^5$?

4. $\dfrac{1}{20} - \dfrac{1}{22} = $ _____

5. Three vertices of a rectangle are the points (1,4), (7,4), (1,8). At which point do the diagonals of the rectangle cross?
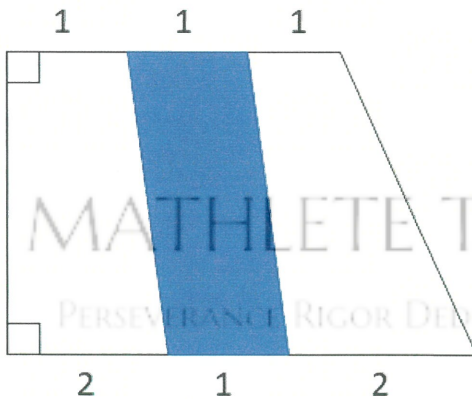

6. What fraction of this trapezium is shaded?

7. The sum of the numbers on these six cards is $4\sqrt{5}$.

$$2\sqrt{7}, \ \sqrt{5}, \ 3\sqrt{5}, \ -4\sqrt{7}, \ -3\sqrt{7}, \ 5\sqrt{7}$$

Lily removes one of them. What is the largest possible sum of the remaining five cards?

8. In this diagram, what is the value of x?
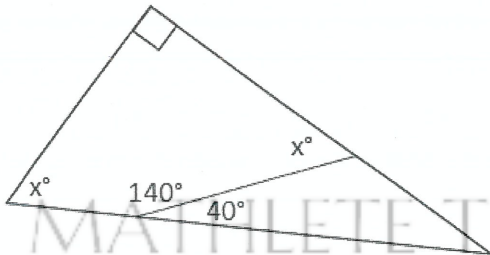
9. The smaller of two dice has three zeroes and three ones on its faces and the larger has the numbers 1, 3, 5, 7, 9 and 11 on its faces.
   Both dice are rolled once and the numbers showing on top are added.
   What is the probability of obtaining a sum of 12?

10. The value of $24^{-\frac{1}{2}} + 17^{-\frac{1}{4}}$ is closest to

   (A) 0.2

   (B) $\dfrac{1}{6}$

   (C) $\dfrac{1}{9}$

   (D) $\dfrac{1}{12}$
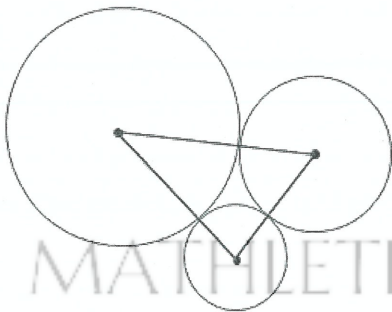
   (E) $\dfrac{1}{36}$

11. The operation * is defined by p * q = 2p - q. Each of p and q is an integer from -6 to +6. How many pairs of values (p, q) will have p * q = q * p?

12. In the diagram, the lengths of the sides of the triangle are 8, 9 and 13 centimetres. The centre of the circles are at the vertices of the triangle and the circles just touch. What is the radius, in centimetres, of the largest circle?
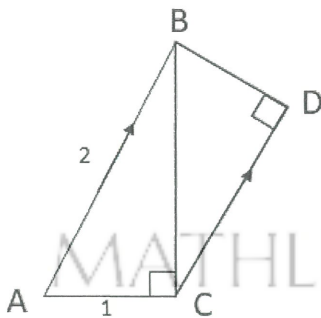
13. The four integers 3, 4, 8, 11 have their mean and range calculated. A fifth integer is then included that is different from the other four. This doesn't change the range, but the mean is now an integer. What is this new mean?

14. Two triangles ABC and BCD are both right angled as shown. Lines AB and CD are parellel. Also $AC = 1$ and $AB = 2$. What is the length of CD?

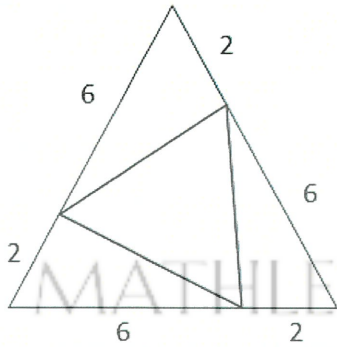15. What is the value of x in this equation?

$$3^x + 3^{x+1} + 3^{x+2} = 13\sqrt{3}$$

16. Four numbers are equally spaced on the number line in the given order

$$\frac{1}{20}, \frac{1}{22}, X, Y$$

What is the value of Y?

17. A small equilateral triangle sits inside a larger equilaterial triangle as shown. What is the ratio of the areas of the smaller and larger equilateral triangles?
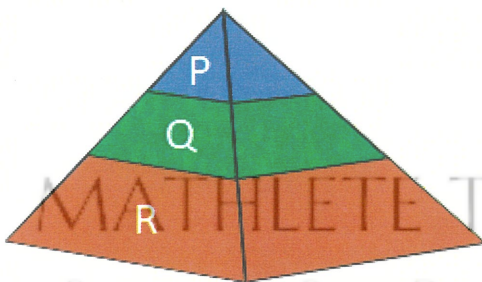
18. The square pyramid shown is divided into 3 pieces, P, Q and R, by two planes that are parallel to the square base. Each of the 3 pieces has the same height. The volume of the piece P is 5cm³.

What is the volume of piece R, in cm³?

19. A sequence of values a_1, a_2, . . . , a_{100} is calculated as follows:

$$a_1 = 1, \ a_2 = 2, \ a_3 = \frac{a_2 + 1}{a_1}, \ \ldots \ a_n = \frac{a_{n-1} + 1}{a_{n-2}}, \ \ldots \ a_{100} = \frac{a_{99} + 1}{a_{98}}$$

What is a_{100}?

20. A triangular ramp is in the shape of a right-angled tetrahedron. The horizontal base is an equilateral triangle with side 8 metres. The apex is 1 metre directly above one corner of the base, so that the two faces are vertical.
In square metres, what is the exact area of the sloping face?
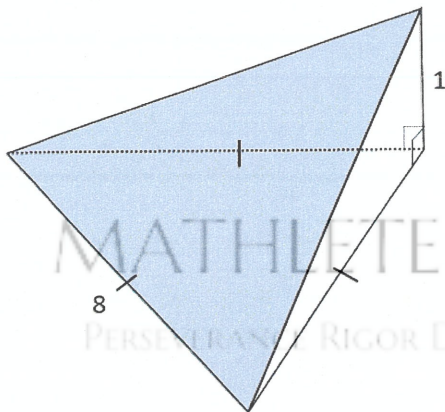
21. A cuboctahedron is a solid formed by joining the midpoints of the edges of a cube as shown. What is the exact volume of a cuboctahedron of side length 2?
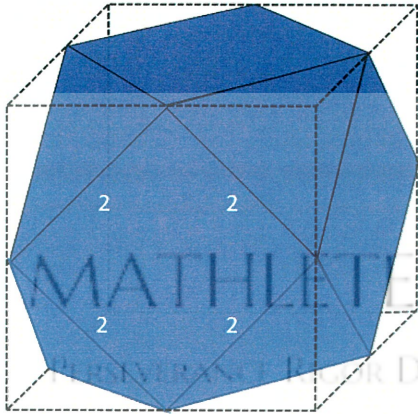

22. In the two equations $ax - b = c$ and $dy + e = f$, each of the letters a, b, c, d, e, and f is replaced by a different digit from 1 to 9.
    When the two equations are solved for x and y, what is the lowest possible value of $x + y$?

23. A semicircle is inscribed in a right-angled isosceles triangle and a square is inscribed in the semicircle as shown. What is the exact ratio of the area of the square to the area of the triangle?
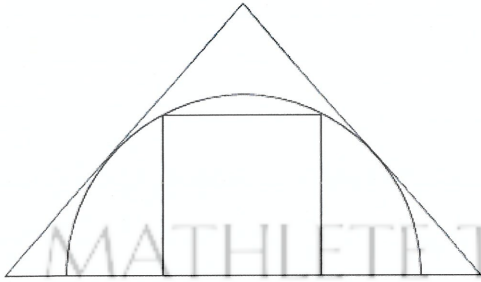
24. In the grid shown, the numbers 1 to 8 are placed so that when joined in ascending order they make a trail. The trail moves from one square to an adjacent square but does not move diagonally. In how many ways can the numbers 1 to 8 be placed in the grid to give such a trail?
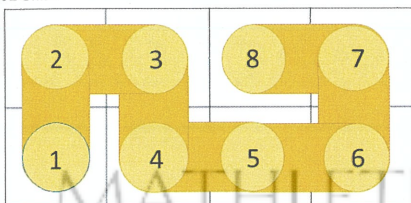
25. When I cycled around the lake yesterday, my children Sally and Wally decided to ride the same route in the opposite direction. We all set off at the same time, from the same point, and finished at that exact same spot. We each rode at our own steady speed.

    It took me 77 minutes. Sally and I passed each other, waving, exactly 42 minutes after we started. Precisely 2 minutes later, Wally and I passed each other, puffing.

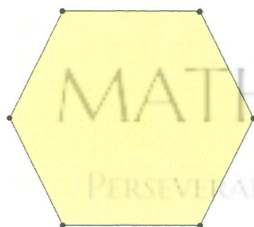   To the nearest minute, how much longer did Wally take than Sally to ride around the lake?

26. Horton has a regular hexagon of area 60. For each choice of three vertices of the hexagon, he writes down the area of the triangle with these three vertices.

   What is the sum of the 20 areas that Horton writes down?

27. An even square number is multiplied by an odd cube greater than 1, resulting in a fourth power. If the fourth power is as small as possible, what is the sum of the square and the cube?

28. In an infinite sequence, the first two terms are 2 and 6, and apart from the first term, each term is one less than the average of its two neighbours. What is the largest term less than 1000?

29. Wasteful Wayne takes one sheet of paper with 'My Document' printed on it. He runs it through the photocopier to make two copies which he then stamps with his 'COPY' stamp.

    Wayne then takes the original and the two copies, runs all three through the photocopier to make two copies of each, stampes the six new copies with his 'COPY' stamp, and adds them to the top of the pile.

    He repeats this process by making two copies of each sheet of paper in his existing pile, stamping the new copies, then adding them to the pile. So the pile triples in size each time. After Wayne has done this eight times in total, the pile is 6561 sheets high.

    How many sheets have exactly 2 'COPY' stamps on them?
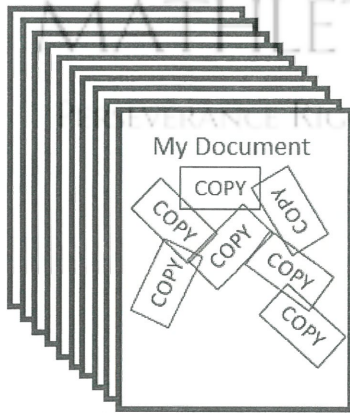
30. Ali, Beth, Chen, Dom and Ella finish a race in alphabetical order: Ali in first place, then Beth, Chen, Dom, and Ella. The next week, they run another race and their placings all change. Two of the runners receive a placing higher than the week before, and the other three runners receive a placing lower than the week before. Given this information, in how many orders could the five runners have finished this second race?