Mathlete Training Centre
SMOPS 2020

1. An apple is placed inside one of the three boxes.
   On Box A, it is labelled "The apple is in this box".
   On Box B, it is labelled "The apple is not in this box".
   On Box C, it is labelled "The apple is not in Box A".
   Given that only one of the statements labelled on the boxes is true, which box is the apple in?
   (1) Box A (2) Box B (3) Box C

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2. Each of the letters A, B, C, D represents a different number in the set $\{1,2,3,4\}$. It is given that $\frac{A}{B} - \frac{C}{D} = 1$, find $B + D$.

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3. It is given that the product of four consecutive natural numbers is 1680. Find the sum of these four numbers.

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4. What is the angle (in degrees) formed by the hour hand and minute hand at 5.40 pm?

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5. The area of rectangle $ABCD$ is 120cm^2. E is the midpoint of BC. F is the midpoint of CD. Find the area of triangle AEF in cm^2

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6. Find the maximum number of regions that 4 circles can divide a plane into. For example, 2 circles can divide a plane into 4 regions.

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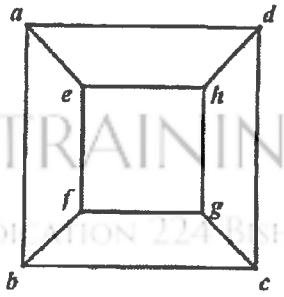
7. What is the largest three-digit prime number with all its digits prime?

8. Given that a polygon with all its angles less than 180° has n acute angles, find the sum of maximum and minimum possible values of n.

9. In the diagram, the eight vertices represent the numbers a, b, c, d, e, f, g, and h respectively. Every number is equal to $\frac{1}{3}$ of the sum of the three adjacent numbers. For example, $a = \frac{1}{3}(b + d + e)$. Find the largest possible value of $(a + b + c + d) - (e + f + g + h)$.



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10. A seven-digit positive integer $\overline{2020ABC}$ is divisible by 7, 8 and 9. How many such integers are there?

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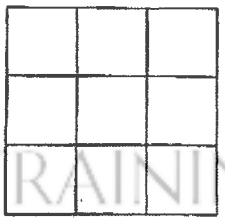
11. Some 50-cent and one-dollar coins are used to fill up a 3×3 board. Each coin is to occupy a 1×1 square on the board. The total value of the coins used is 6 dollars. Given that the total value of the coins in every 2×2 square is a dollars, find the largest possible value of a.



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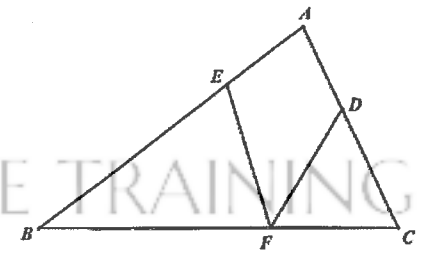
12. In triangle ABC, $\angle BAC = 80°, BE = BF$ and $CD = CF$. Find the size of $\angle DFE$ in degrees.



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13. It is given that $\underbrace{111...111}_{n \text{ digits}}$ is divisible by $\underbrace{333...333}_{100 \text{ digits}}$. Find the least possible value of n.

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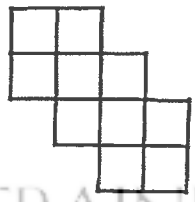
14. In the diagram below, we wish to fill out the 10 small squares using numbers $1, 2, 3, 4, 5, 11, 12, 13, 14, 15$ without repetition, such that the sum of four members in every 2×2 square is equal. Find the largest possible value of the sum.



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15. In 2019, a smartphone sold at 80% of its marked price made a profit of 20% on its cost. In 2020, the cost of the same model is lowered by 100 dollars. It is known that if the retailer sells the phone at 75% of the same marked price, he would actually make a 25% profit. What is the cost of this smartphone in 2020?

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16. Find the sum of reciprocals of all positive factors of 120. The reciprocal of a number a is defined as $\frac{1}{a}$.

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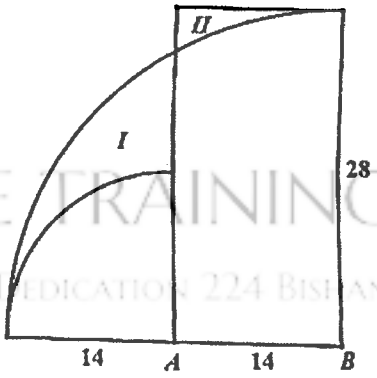
17. In the diagram, A and B are the centres of two quarter-circles of radii 14 cm and 28 cm respectively. Find the difference between areas of region I and II in cm^2. (Take π to be $\frac{22}{7}$).



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18. Let x and y be positive integers less than 100. Find the number of ordered pairs (x, y) such that x^2+y^2 is divisible by 49. (Note:(1,2) and (2,1) are considered two different ordered pairs.)

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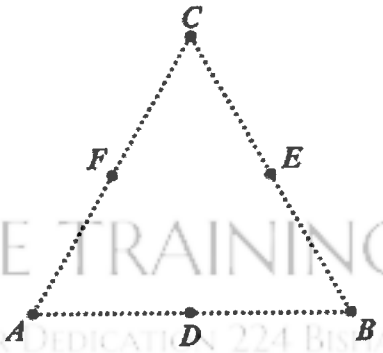
19. If $\frac{20}{13}$ were to be written in its decimal representation, what is the sum of the first 2020 digits after the decimal point?

20. In the diagram, points A, B and C are vertices of an equilateral triangle, and points D, E and F are midpoints of the sides. How many non-congruent triangles can be drawn using any three of these six points as vertices?
(Note: two triangles are congruent if they can overlap exactly. For example, triangle ADF and triangle DFE are congruent. Triangle ADF and triangle CED are non-congruent.)



21. A pair of two *consecutive* numbers is called a "buddy" pair if there is no carry over in any digits when addition is performed. For example, 134 and 135 is a "buddy" pair, but 135 and 136 is not. In the list of integers 1001, 1002, ..., 1999, 2000, how many "buddy" pairs are there?

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22. Given that $x = 20182019 \times 2020, y = 20182020 \times 2019$ and $z = 20202019 \times 2018$, which of the following is correct?
(1)$z<y<x$ (2)$z<x<y$ (3)$x<y<z$ (4)$x<z<y$ (5)$y<z<x$

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23. Find the sum of all possible products of a prime number and a composite number chosen from the following numbers: $2, 3, 4, 5, ..., 20$. (Note: Composite numbers are non-prime numbers greater than 1. Also take note that if a product appears more than once, for example, 12 can be obtained as 2×6 or 3×4, then 12 needs to be added twice when the sum is computed.)

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24. Three generous friends, each with some money, redistribute the money as follows:
Richard gives enough money to Simon and Timothy to double their amount.
Simon then gives enough money to Richard and Timothy to double their amount.
Finally, Timothy gives enough money to Richard and Simon to double their amount.
If Timothy had 36 dollars at the beginning and 36 dollars at the end, what is the total amount that all three people have?

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25. How many digits are there when the product $\underbrace{2 \times 2 \times ... \times 2 \times 2}_{100\text{ '2's}}$ is evaluated? (For example, when $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$ is evaluated, the result 128 is a 3-digit number.)

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26. Two cities are 999km apart. There are 1000 distance signs along the higway connecting the cities at every 1km mark. The signs are in the following format: (0,999),(1,998),(2,997),...,(998,1),(999,0). We call a distance sign "nice" if it contains only two distinct digits. For example, (0,999) is "nice" as it only contains 0 and 9. How many "nice" distance signs are there?

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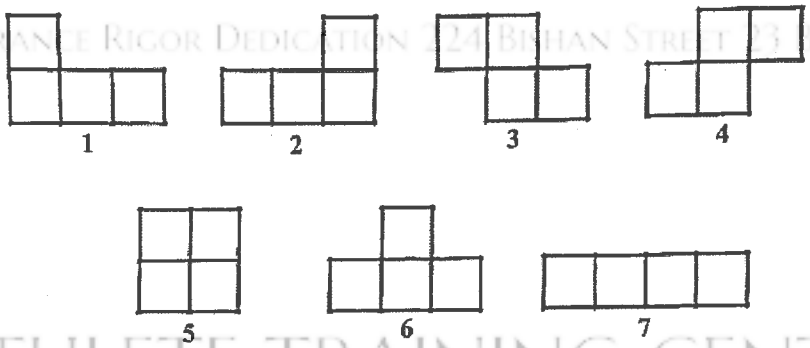
27. How many ways are there to arrange $1, 2, 3, 4, 5$ in a line such that every digit is not greater than the average of the two neighbouring digits? Note that this condition does not apply to the leftmost and rightmost digits as they only have adjacent digit on one side.

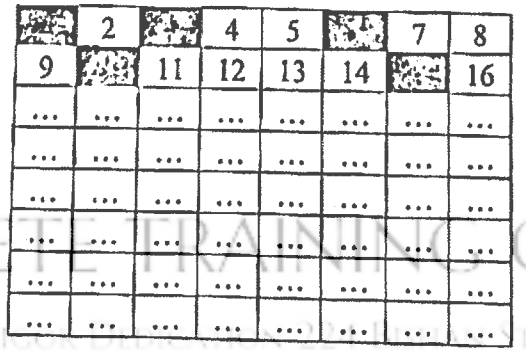
28. There are 7 types of tetrominoes as shown in the diagram below. They are labelled from 1 to 7.



Mr Tetris used four *distinct* tetrominoes (rotation is allowed) to form a square of area 16 square units. If the sum of the four labels is calculated, what is the least possible value of this sum?

29. How many different ways are there to express 10000 as the product of 3 positive integers? (Note: order is important, for example, $1 \times 100 \times 100$ and $100 \times 1 \times 100$ are considered two different ways.)

30. A rectangular board with 8 columns has its squares numbered as follows: the first row is numbered 1 to 8, the second row is numbered 9 to 16, and so on. A student shades square 1, then skips one square and shades square 3, skips two squares and shades square 6, and continues in this way until there is at least one shaded square in each column. What is the number in the final shaded square?



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