Mathlete Training Centre

SMOPS 2019

1. If $\overline{ab} + \overline{ba} + b = \overline{aab}$, find $a + b$.
   ($\overline{ab}$ denotes a 2-digit number where the tens digit is a and the unit digit is b.)

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2. Given a and b are positive integers (whole numbers excluding 0), how many ordered pairs of numbers (a, b) are there such that

$$a^2 + b^2 \leq 14 \ ?$$

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For example, (1, 2) and (2, 1) are considered two different ordered pairs.

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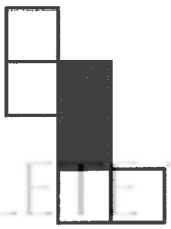
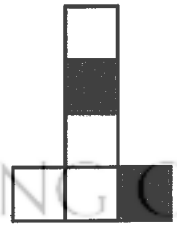
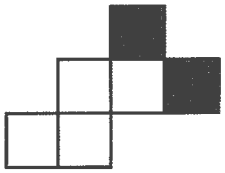
3. When these nets are folded to make cubes, which (if any) will have the two shaded faces directly opposite each other?



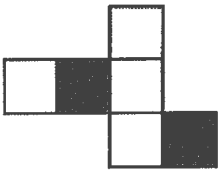
A



B



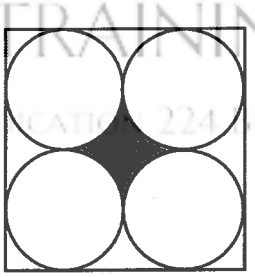
C



D

E. None of the given nets.

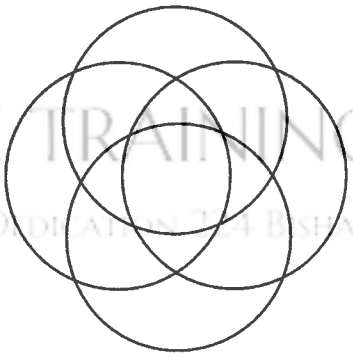
4. In the diagram below, four identical circles are drawn inside a square of area 1600 cm^2. Find the area of the shaded region. Take $\pi = 3.14$.



5. In a football tournament, there are k teams.
   Each team plays against every other team exactly once.
   Three points are awarded to a team for a win; two points for each of the two teams in a draw; one point for a loss.
   At the end of the tournament, the total points of all the teams are 24.
   Find the value of k

6. Given $\dfrac{79}{137} = a - \cfrac{1}{b + \cfrac{1}{c - \cfrac{1}{d + \cfrac{1}{e}}}}$,
   where a, b, c, d, e are whole numbers, find the value of $a + b + c + d + e$.

7. In the diagram below, each of the integers from 1 to 13 is filled in one of the regions created by the four circles. Each circle contains seven numbers.
The sum of the numbers in each circle is S_1, S_2, S_3, S_4.
Find the largest value of $S_1 + S_2 + S_3 + S_4$



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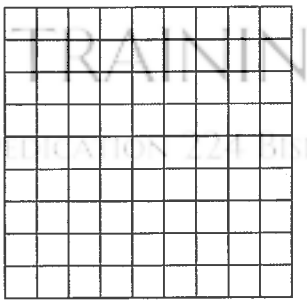
8. In a 9×9 square grid, exactly 29 of the unit squares are shaded. Other cells are white.
All the rows and columns have at least one unit square shaded each.
There are x columns and y rows with at least five shaded cells each.
Find the largest value of $x + y$.



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9. In a test, there are ten multiple choice questions.
Four points are awarded for a correct answer. One point is deducted from the total for every wrong answer. No point is given for an unanswered question.
How many different total points can the student score in this test?

10. Consider the table below where the numbers are arranged according to a pattern:

	1	3	5	7
15	13	11	9	
	17	19	21	23
31	29	27	25	
	...	...	...	...

The number 9 is in row 2 and column 4.
If the number 2019 is in row m and column n, find the value of $m+n$.

11. A number N is divisible by each of the integers 2, 3, 4, 5, 6, 7, 8 and 9.
N gives a remainder of 5 when divided by 7.
Find the smallest value of N.

12. On a circular track of circumference 20 m, a robot A travels anticlockwise at a constant speed of 3.4m/s, while another robot B travels clockwise at a constant speed of 1.5m/s. They both start at the same point and at the same time.
At most how many different points on the track will the two robots pass each other?

13. Find the sum of digits in the number

12345678910111213...9998999,

14. A bag contains blue, white and red marbles.
The number of blue marbles is at least equal to half the number of white marbles, and at most equal to one third of the red marbles.
Given that the sum of the white marbles and the blue marbles is at least 55.
At least how many red marbles are there?

15. Arrange the three numbers below from the smallest to the largest:

$$x = \frac{1}{2} \times \frac{3}{4} \times \frac{5}{6} \times ... \times \frac{47}{48}, \quad y = \frac{2}{3} \times \frac{4}{5} \times \frac{5}{6} \times ... \times \frac{48}{49}, \quad z = \frac{1}{7}$$

16. Find the 2019^{th} digit in the number

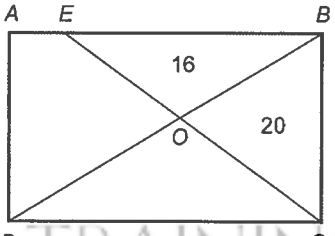
12345678910111213...998999.

17. In how many ways can we choose two numbers from

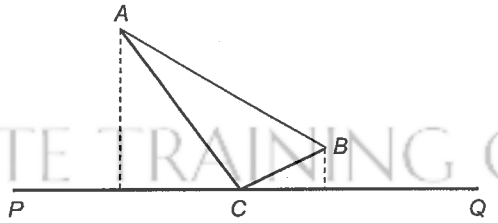
$$19, 20, 21, 22, ..., 78, 79$$

such that the sum of two numbers is even?

18. In a rectangle $ABCD$, the areas of two triangles are given.
If $AE = \frac{1}{5}AB$, find the area of quadrilateral $ADOE$.



19. In the diagram below, $AB = 3m$ and $PQ = 8m$. The perpendicular distances from A, B to the line PQ are 2m and $\frac{1}{2}$ m respectively. A point C varies on the line PQ. Find the largest value of $AC - CB$.



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20. Among the integers: 1, 2, 3, ..., 49, 50, what is the maximum number of integers that can be selected such that the sum of any two selected numbers is not divisible by 7?

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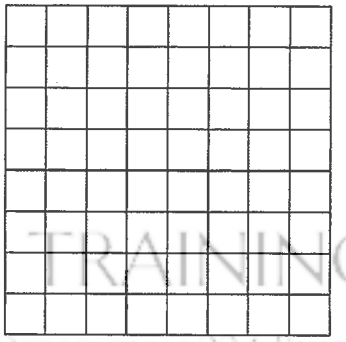
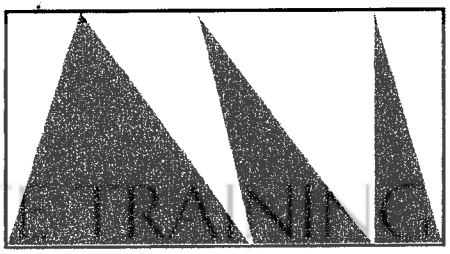
21. Evaluate $\dfrac{(2^2+4^2+6^2+...+100^2)-(1^2+3^2+...+99^2)}{50}$.

22. Given an ordinary 8 by 8 square chessboard as shown, find the number of different ways of choosing one piece of  which is made up of four square units.



23. The diagram shows a rectangle with dimension 15 cm by 8 cm. Find the total area of shaded region in cm^2.



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24. Find the value of $(2019 + 101) \times (2019 + 99) - (200 + 2019) \times 2019$.

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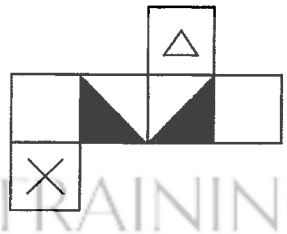
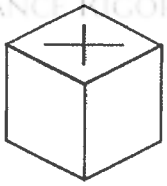
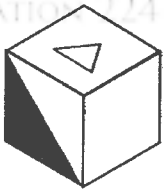
25. A piece of paper as shown below can be folded into a cube. Which of the following options shows the correct cube?





(1)



(2)

(3)

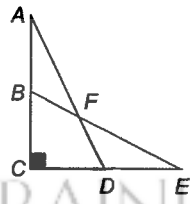
26. In the diagram, the shaded region outside the circle occupies $\frac{1}{6}$ of the circle and $\frac{1}{5}$ of the square. On the other hand, the shaded region inside the triangle occupies $\frac{1}{9}$ of the triangle and $\frac{1}{4}$ of the square. If the area ratio of circle : square : triangle is $24 : a : b$, find $a + b$.

27. There are two cards in a bag. One card has both sides red whereas the other card has one side red and one side blue. While blindfolded, a person randomly draws one card from the bag and places the card on the table. If the side facing up is red, what is the possiblity that the side facing down is also red?
Choose your answer from the following options.

(1) $\frac{1}{4}$ (2) $\frac{2}{3}$ (3) $\frac{3}{4}$ (4) 1

28. It is given that 1059, 1417 and 2312 gives the same remainder r when they are divided by the same divisor d, where $d > 1$. Find the value of $d - r$.

29. In the diagram, AC is perpendicular to CE. Point B is the midpoint of AC. Point D is the midpoint of CE. AD and BE intersect at point F. if $BC = CD = 30$cm, find the area of triangle DEF in cm^2.



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30. A three-digit number is divisible by the sum of its digits and its quotient is equal to q. Find the last possible value of q.

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31. In the diagram, points F, A, C are on the same straight line. Points A, B, G, are on the same straight line. Point D is on the extension of CB such that $\angle FAD = \angle BAD$ and point E is on the extension of AC such that $\angle CBE = \angle GBE$. It is known that $AD = AB = BE$. Find the measure of $\angle BAC$ in degrees.