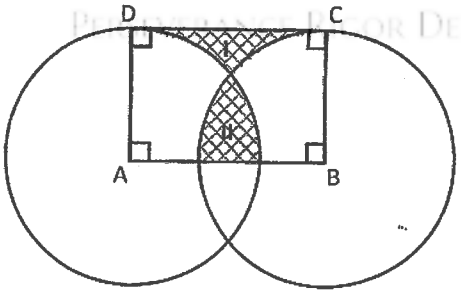
Mathlete Training Centre
SMOPS 2018

1. Find $63 \div 34 \times 51 \div 24 \times 64 \div 36$.

2. Flights between Singapore and City A take 11 hours. A flight leaves City A at 3am, and reaches Singapore at 9pm the same day. If the flight leaves Singapore at 3am, what time will it arrive at City A? (All timings given are local time).
   (1) 8pm (2) 9pm (3) 10pm (4) 7am (5) 8am (6) 9am

3. Which of the following numbers is a perfect square?
   (1) 921438 (2) 2660161 (3) 76168 (4) 750235

4. In the figure below, two equal circles have radii $AD = BC = 20$cm. If the regions **I** and **II** have the same area, determine the area of $ABCD$. Take $\pi = 3.14$.



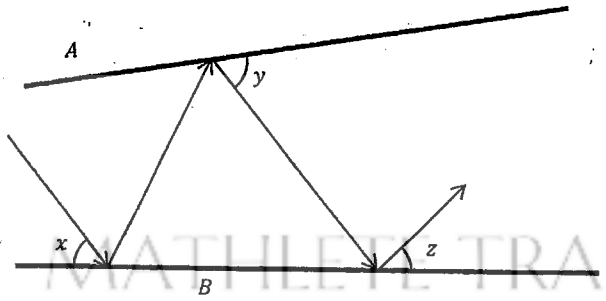
5. Find a 4-digit number which is equal to the fourth power of the sum of its digits.

6. Between the row of numbers below, we can add 5 '+' signs to make a sum. For example, $123+4+5+67+8+9$. Suppose the maximum possible sum is a and the minimum possible sum is b. Determine the value of $a-b$.

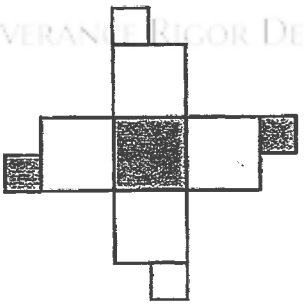
7. If we write $\frac{5}{14}$ as a decimal, what is the 203^{rd} digit after the decimal point?

8. A bag contains several red and white balls. If we remove 1 red ball and 2 white balls and do this repeatedly, we are left with 5 red balls. If we remove 3 red balls and 4 white balls repeatedly, we are left with 6 white balls. How many balls were originally in the bag?

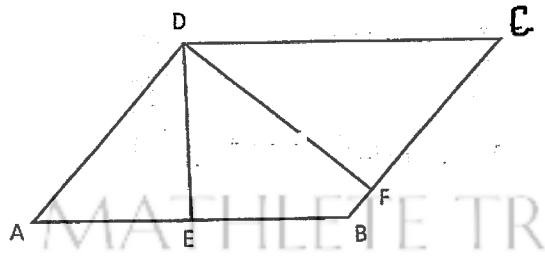
9. A light ray reflects between 2 mirrors A and B, as shown in the figure below. Given that $\angle x = 60°, \angle y = 50°$, find $\angle z$.



10. The below net can be folded into a cube. Which of the following is a correct depiction of the cube?



11. In the adjacent figure, parallelogram $ABCD$ has perimeter 100 cm. DE and DF are perpendicular to AB and BC respectively. Suppose that $DE = 8$cm, $DF = 12$ cm. Find the area of $ABCD$.



12. Consider the sequence $1, 2, 3, 2, 3, 4, 3, 4, 5, 4, 5, 6, 5, 6, 7, 6, 7, 8, 7, 8, 9, \ldots$. What is the sum of the first 100 terms of this sequence?

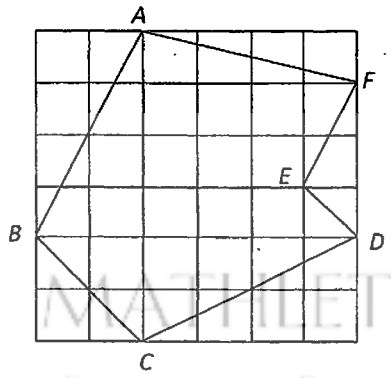
13. Find the minimum whole number N that satisfies the following conditions: N can be written as the sum of 9 consecutive whole numbers, 10 consecutive whole numbers and 11 consecutive whole numbers.

14. There are some 4-digit numbers that satisfy the following: if you multiply their digit sum by 10, and subtract this from the original number, you get exactly 3015. What is the largest such 4-digit number?

15. Among whole numbers between 1 to 100000, how many are divisible by 15 and 18 but not 28?

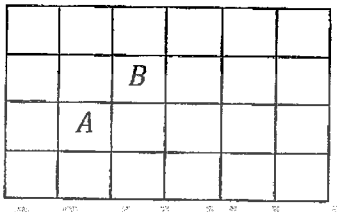
16. Given that $A = 1 + \frac{1}{2^2} + \frac{1}{2^4} + \frac{1}{2^6} + \cdots + \frac{1}{2^{100}}$, $B = 1 + \frac{1}{2} + \frac{1}{2^3} + \frac{1}{2^5} + \cdots + \frac{1}{2^{99}}$, which of the following is true about the relationship between A and B?
(1) $A = 2B$ (2) $B = 2A$ (3) $A = 2B - 1$ (4) $B = 2A - 1$ (5) $B = 2A - 2$

17. In the figure below, each small square has area 4 cm^2. Find the area of hexagon $ABCDEF$.

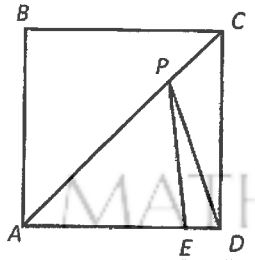


18. Given that 16 distinct positive integers sum to 2018, what is the maximum possible value of the 8^{th} smallest integer?

19. In the figure below, a 4×6 grid contains 2 squares marked A and B. How many rectangles in the figure contain these 2 squares in their interior?

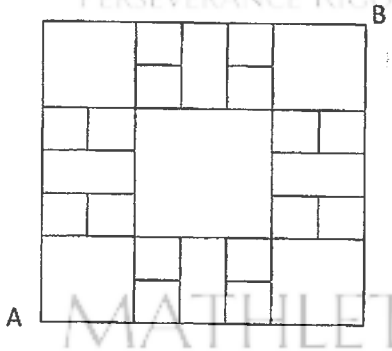


20. In the figure below, P is a point on diagonal AC of square $ABCD$. E lies on AD such that $AE = 3\text{cm}$, $ED = 1\text{cm}$. Find the maximum value of $PD + PE$.



21. A palindrome is a positive whole number that reads the same forwards and backwards. For example, 3, 22, 505 and 8338 are all palindromes. If we arrange all palindromes in ascending order, what is the 400^{th} palindrome?

22. In the figure below, how many ways are there to travel from A to B, only moving upwards or rightwards along the gridlines?



23. How many 4-digit numbers contain at least 1 digit '4', but are not divisible by 4?

24. Find the smallest n such that $\underbrace{2018201820182018985}_{n\text{'2018's}}$ is a multiple of 11.

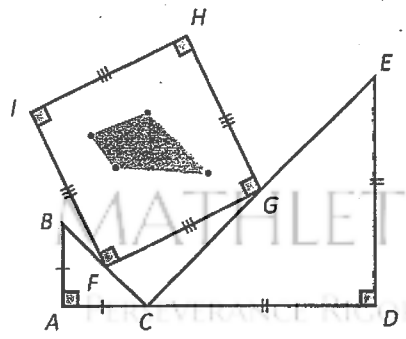
25. What is the value of:

$$\cfrac{15}{2+\cfrac{1}{2+\cfrac{1}{3+\cfrac{1}{2018+\cfrac{1}{2019}}}}}+\cfrac{15}{1+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{3+\cfrac{1}{2018+\cfrac{1}{2019}}}}}}$$

26. Using each of the digits $1, 2, 3, \ldots, 9$ exactly once, at most how many prime numbers can be formed? For example, 479, 281 and 653 are 3 primes numbers formed in this manner.

27. A and B are running on a straight 100m track. A needs 15 seconds to run from one end to another, while B needs 20 seconds. If they begin running at a constant speed starting from the same end, how many times will they pass each other in 2 minutes?

28. In the figure below, isosceles right-angled triangles ABC and CDE have sides $AC = 2$cm, $CD = 6$cm. F, G are midpoints of BC, CE respectively. $FGHI$ is a square constructed on side FG. Given that the shaded area in the square has the same area as isosceles right-angled triangle ABC, find the unshaded area inside $FGHI$.



29. If $S = \frac{(1+2)^2}{1\times 2} + \frac{(2+3)^2}{2\times 3} + \frac{(3+4)^2}{3\times 4} + \cdots + \frac{(99+100)^2}{99\times 100} + \frac{(100+101)^2}{100\times 101}$, find $101S$.

30. How many ways can we pick 2 distinct numbers out of the set $\{19, 20, 21, \ldots, 97, 98, 99\}$ such that their product is divisible by 6? (Note: the order of the 2 numbers does not matter.

END OF PAPER

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