Mathlete Training Centre
2015 Questions

1. (SMOPS 15Q1) On a 200 metre circular track, A and B start running from the same position at the same time, going in a clockwise direction. A runs 6 metres per second, and B runs 4 metres per second. How many times does A **overtake** B within the next 16 minutes?

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2. (SMOPS 15Q2) The ratio of the length, width, and height of a cuboid is 4:3:2. The total length of all the edges is 72 cm. Find the volume of this cuboid.

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3. (SMOPS 15Q3) Find the value of $(101 + 234 + 567) \times (234 + 567 + 89) - (101 + 234 + 567 + 89) \times (234 + 567)$.

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4. (SMOPS 15Q4) In a bag, there are 10 red balls, 10 yellow balls, and 10 green balls. These balls have the same size; the red balls are marked with a number '4', yellow balls are marked with a number '5', and green balls are marked with a number '6'. Claire randomly takes 8 balls out of the bag, and their sum is 39. What is the maximum possible number of red balls among these 8 balls?

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5. (SMOPS 15Q5) A is a positive integer that satisfies the following condition: $\frac{5}{9} < \frac{9}{A} < 1$. How many different values can A take?

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6. (SMOPS 15Q6) A certain month has 31 days, and the number of Mondays is equal to the number of Fridays. What day is the 10^{th} day of the month?

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7. (SMOPS 15Q7) The last four digits of the product of a positive integer with 411 is '2015'. What is the minimum possible value of this positive integer?

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8. (SMOPS 15Q8) As shown, a football is made of 32 pieces of leather. The white pieces are regular hexagons, and the black pieces are regular pentagons. Each white piece is neighbours with 3 other white pieces and 3 black pieces; each black piece is neighbours with 5 white pieces. How many white pieces are there?

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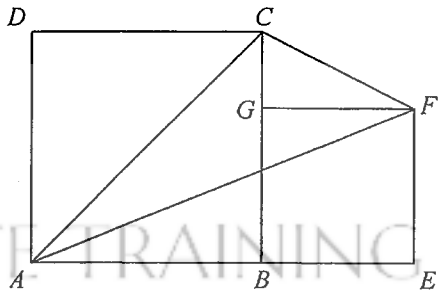
9. (SMOPS 15Q9) As shown, square ABCD has side length 8 cm. E is on AB extended, and BEFG is a square. Find the area of ΔAFC.



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10. (SMOPS 15Q10) There is a promotion: every 5 empty soda bottles can be exchanged for 1 full soda bottle. In a class party, 109 bottles were drank, some of which were exchanged using the promotion above. What is the minimum possible number of full soda bottles that they bought?

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11. (SMOPS 15Q11) The following figure can be folded along the solid lines in order to form a polyhedral. How many edges will this polyhedral have?

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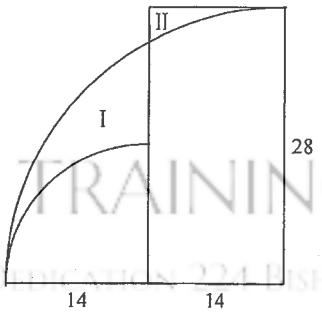
12. (SMOPS 15Q12) 2^{29} is a nine-digit number made of 9 different digits. Which digit does not appear?

13. (SMOPS 15Q13) A class has 52 students, of which 30 can swim, 35 can ride a bicycle, 42 can play table tennis. At least how many students in this class can do all 3 activites?

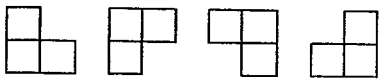
14. (SMOPS 15Q14) A transport company needs to transport 89 tons of cargo from A to B. The big truck can carry 7 tons of cargo, the small truck can carry 4 tons. If every big truck uses 14 litres of fuel on the journey from A to B, and the small truck uses 9 litres, what is the minimum amount of fuel needed to transport these cargo?

15. (SMOPS 15Q15) In a convex polygon, the sum of all internal angles, excluding one, is 2015°. Find the angle of the excluded internal angle.

16. (SMOPS 15Q16) As shown, A and B are the centres of two quarter circles, of radius 14 and 28 cm, respectively. Find the difference in the areas I and II, as marked on the figure. Use $\pi = \frac{22}{7}$.



17. (SMOPS 15Q17) In a 6×6 grid, how many L shaped figures (shown below) are there?



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18. (SMOPS 15Q18) Two natural numbers, A and B, are such that when A is divided by B, the quotient is 15 and remainder is 5. The sum of A, B, quotient, and remainder is 2169. Find A.

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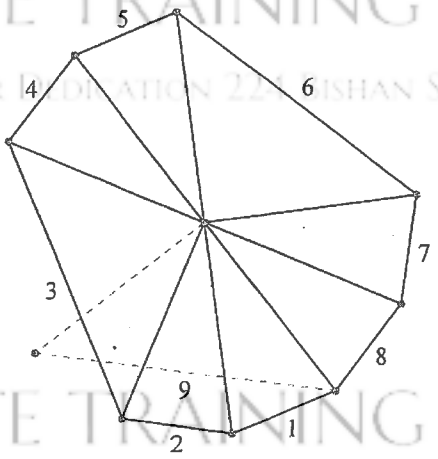
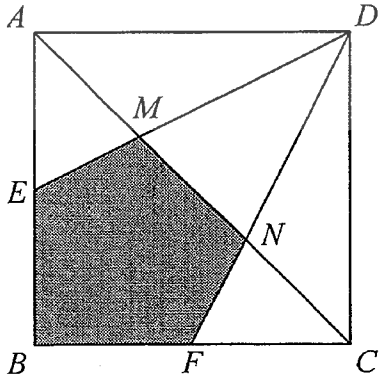
19. (SMOPS 15Q19) In the figure below, there are two types of isosceles triangles with sides length 1. Type A makes an angle of 30° at the tip, and type B is a right-angled isosceles triangle. As shown, we now place these two types of triangles around a point in a clockwise manner. The 1st and 2nd are type A, the 3rd is type B, the 4th and 5th are type A, the 6th is type B, and so on. If the n^{th} triangle exactly overlaps with the 1st triangle (that means the n^{th} triangle also has to be a type A), find the minimum value of n.

In the figure, the 9^{th} triangle starts with the same edge with the 1st triangle, but it does not satisfy the condition because it is a type B triangle.



20. (SMOPS 15Q20) As shown, ABCD is a square, E and F are the midpoints of AB and BC. DE and DF intersects the diagonal AC at points M and N. If the area of square ABCD is 48 cm^2, find the area of the shaded part EBFNM.



21. (SMOPS 15Q21) Four football teams are in a round robin competition, where they have a match between every 2 teams. The winning team gains 3 points, the losing team gains 0 points, and each team gets 1 point if the match ends in a draw. After all the matches, the scores of each team are 4 consecutive numbers. Find the product of these 4 numbers.

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22. (SMOPS 15Q22) A pool has 5 pipes, A, B, C, D, E. When only A, B, C, D are opened, they need 6 hours to fill the pool. When B, C, D, E are opened, they need 8 hours to fill the pool. If only A and E are opened, they need 12 hours to fill the pool. If we only use pipe E, how many hours does it need to fill the pool?

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23. (SMOPS 15Q23) How many consecutive '0's are there at the end of $5 \times 10 \times 15 \times 20 \times ... \times 2010 \times 2015$?

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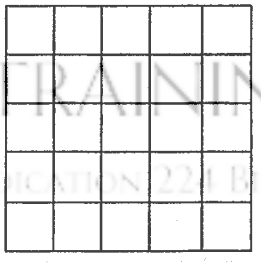
24. (SMOPS 15Q24) How many cells do we need to colour, in order that any 3×3 square contains exactly 4 coloured cells?



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25. (SMOPS 15Q25) From 49 balls, marked with numbers 1, 2, ..., 49, we pick n balls and arrange them into a circle, such that the product of any two neighbouring numbers is less than 100. What is the maximum value of n?

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26. (SMOPS 15Q26) Train 1 and 2 departs from stations A and B, travelling towards each other. The ratio of the speeds of train 1 and 2 is 3:2. Station C is the midpoint between station A and B; train A reaches station C at 9 a.m., and train B reaches station C at 7 p.m.. Find the timing when trains 1 and 2 meet each other?

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27. (SMOPS 15Q27) Let a, b, and c be any number from 0 to 9 (they may be identical). If we express all recurring decimal numbers $0.\dot{a}b\dot{c}$ as fractions in their simplest form, how many different numerators are there?
$0.\dot{a}b\dot{c} = 0.abcabcabc...$
hint: consider all numberator that is divisible by neither 3 nor 37, then consider those divisible by 3. Note no numerator is divisible by 37.

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28. (SMOPS 15Q28) Put 5 cards, A, K, Q, J, 10 randomly into 5 envelopes marked A, K, Q, J, 10, such that there is one card in every envelope. How many different ways are there such that every card is placed in the wrong envelope?
hint: the formula for number of derangements with n elements is

$$n! - \binom{n}{1}(n-1)! + \binom{n}{2}(n-2)! - \cdots \pm \binom{n}{n}1!.$$

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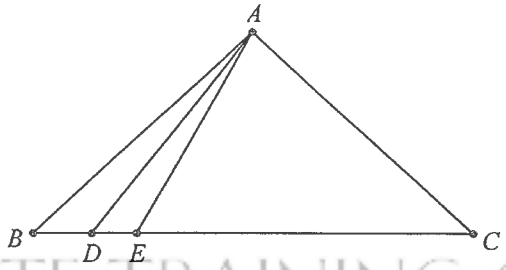
By using PIE, all arrangements of the whole sequence subtract all arrangements in which each element appears in its original location then add back permutations in which each set of two elements stay in their original positions, as we subtracted them twice. Continuing this pattern gives us the formula.

29. (SMOPS 15Q29) How many 5-digit numbers are there that are divisible by 3, and at least one digit is '3'?
hint: for every 4 digit number that has no 3, we have 3 options to put a digit at the right to make a five digit number divisible by 3 and has no digit 3 inside.

30. (SMOPS 16Q30) As shown, B, D, E, C lie on a straight line. $\angle BAD = \angle DAE = 12°$, AC is perpendicular to AD, BC=AB+AE. Find the value of $\angle ABC$.



hint: reflect to make broken line into straight line.

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