Mathlete Training Centre
SMOPS 2013

1. (SMOPS 13Q1) Find the last 5 digits of the sum $127354 + 27354 + 7354 + 354 + 4$

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2. (SMOPS 13Q2) Find the sum of all two-digit numbers whose units digit and tens digit are both even.

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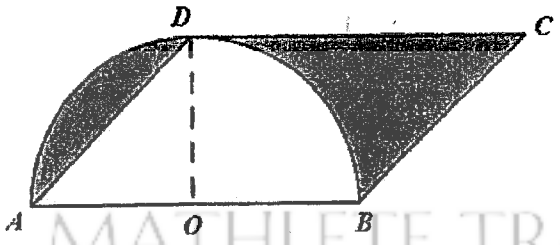
3. (SMOPS 13Q3) The diagram shows a semicircle with centre O overlapped with a paralleogram $ABCD$. The diameter AB of the semicircle is 12 cm. Find the total area of the shaded region in cm^2.



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4. (SMOPS 13Q4) Abel, Ben and Charlie took part in SMOPS 2012, which comprised 30 questions. They had correctly answered 26, 23 and 18 questions respectively. What is the least number of questionss that were correctly answered by all 3 students?

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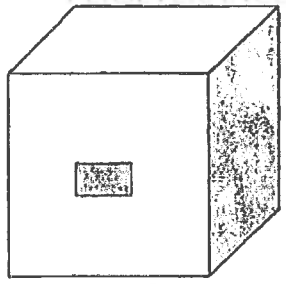
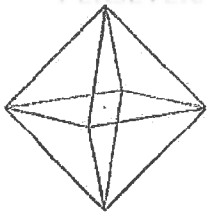
5. (SMOPS 13Q5) Find the value of $555 \times 554555 - 554 \times 555554$.

6. (SMOPS 13Q6) The diagram shows a cube with side length 5 cm. If a rectangular tunnel with dimensions 2 cm by 3 cm is made in the middle of the cube, find the amount increase in the total surface area of the resulting solid in cm^2.

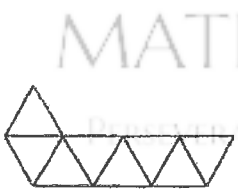


7. (SMOPS 13Q7) The average value of the four whole numbers $\overline{a}, \overline{b5}, \overline{c17}, \overline{d432}$ where a, b, c and d each represents the first digit of a number, is 1735. Find the value of $a + b + c + d$.

8. (SMOPS 13Q8) The diagram shows a regular octahedron, which is a solid composed of eight equilateral triangles.

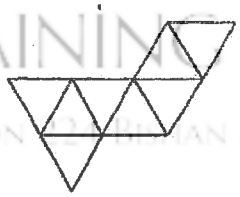


Which of the following patterns can be folded into a regular octahedron?



Pattern (1)

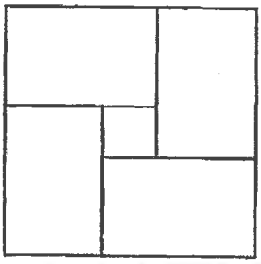
Pattern (2)



Pattern (3)

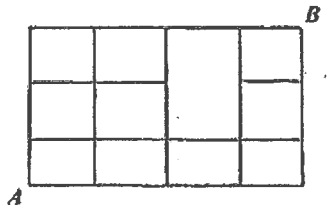
9. (SMOPS 13Q9) The sum of 2 prime numbers is equal to 2013. Find the product of these two numbers.

10. (SMOPS 13Q10) The figure shows a big square, which is divided into four identical rectangles and one small square. Given that the area of the small square is 16 cm^2, the area of each rectangle is 140 cm^2, find the width of each rectangle in cm.

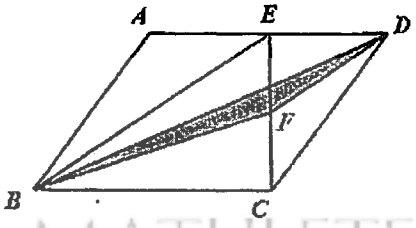


11. (SMOPS 13Q11) A 5-digit number written in the form $\overline{24abc}$ has the last three digits unknown. If this number is divisible by 3, 4 and 5 respectively, find the greatest possible value that $\overline{abc}$ can take.

12. (SMOPS 13Q12) In the diagram on the right, an ant is moving from A to B. If the ant is only allowed to move to the right or upwards along the grid lines, how many different paths are there from A to B?



13. (SMOPS 13Q13) The diagram shows a parallelogram $ABCD$. E is the midpoint of AD. F is the midpoint of EC. If the area of the triangle BFD is 9 cm^2, find the area of the parallelogram $ABCD$ in cm^2.



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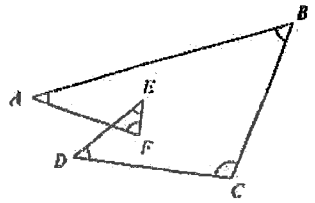
14. (SMOPS 13Q14) If we write $\frac{2013}{1990}$ in the form

$$a+\cfrac{1}{b+\cfrac{1}{c+\cfrac{1}{d+\cfrac{1}{e}}}},$$

where a, b, c, d and e are positive integers, what is the value of $a+b+c+d+e$?

15. (SMOPS 13Q15) In the given diagram, find the value (in degrees) of $\angle A + \angle B + \angle C + \angle D + \angle E + \angle F$.



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16. (SMOPS 13Q16) In how many different ways can four children share 8 identical chocolates so that each child gets at least one?

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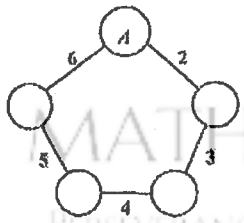
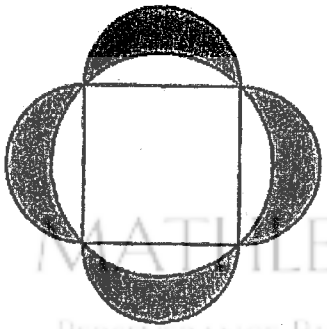
17. (SMOPS 13Q17) In the given diagram, each circle contains a natural number and the diagram satisifies the following conditions:
i) The number labeled along each edge represents the difference between the numbers in the two circles joined by the edge.
The sum of the numbers in the 5 circles is equal to 1979.
Find the number in circle A.



18. (SMOPS 13Q18) A certain type of water bottle is sold at \$10 in both Store A and Store B. Mrs Lim would like to buy a few water bottles for a Chilren's Home. Store A sells the water bottle with an offer of "Buy 5 Get 1 Free" (no free bottle for buying fewer than 5 water bottles); Store B gives a 15% discount for customers who buy 4 or more water bottles. What is the least amount of money that Mrs Lim needs to spend in ordr to get 14 water bottles?

19. (SMOPS 13Q19) A square of side lenth 18 cm is inscribed in a circle. Semicircles are constructed on its sides, as shown in the diagram. Find the total area of the four shaded lunes in cm^2.



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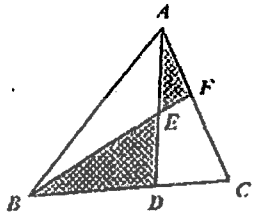
20. (SMOPS 13Q20) Four teams participated in a soccer tournament. Each team played against all other teams once each. 3 points were awarded for a win, 1 point for a draw and 0 point for a loss. At the end of the tournamnet, the four teams have obtained 5, 1, x, 6 points respectively. Find the value of x.

21. (SMOPS 13Q21) In the diagram, the area of the triangle ABC is 40. Given that $2BD = 3CD$ and $AE = DE$, find the area of the shaded region.



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22. (SMOPS 13Q22) If integers are selected randomly from 1 to 35, what is the minimum number of integers that need to be selected such that among the chosen numbers we can always find two integers whose difference is 7?

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23. (SMOPS 13Q23) A team of workers are sent to two construction sites A and B respectively. The amount of work to be done at construction site A is 50% more than that at construction site B. In the morning, the number of workers sent to construction site A is 3 times the number of workers sent to site B. In the afternoon, the ratio of workers sent to construction sites A and B is 7 : 5. By the end of the day, the work at construction site A is fully completed, while construction site B stilll requires 8 workers to work for another full day. Assuming the workers work at the same rate, find the total number of workers in this team.

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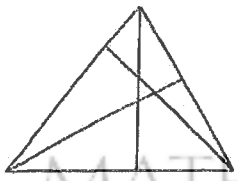
24. (SMOPS 13Q24) Find the total number of triangles in the diagram.



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25. (SMOPS 13Q25) A car and a motorcycle started travelling towards each other at the same instant, from cities A and B respectively. 72 minutes later, they met along the road and contiued to travel towards their destinations. Given that the speed of the car is $1\frac{1}{3}$ times that of the motorcycle, how many minutes after the car reached city B would the motorcycle reach city A?

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26. (SMOPS 13Q26) Given four prime numbers a, b, c and d, if the product of $a \times b \times c \times d$ is the sum of 55 consecutive positive integers, find the smallest possible value of $a + b + c + d$.

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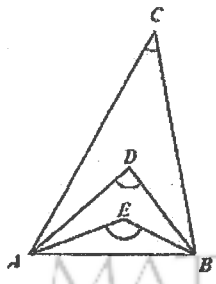
27. (SMOPS 13Q27) A particular brand of car tyre lasts 300 km on a front wheel or 450 km on a rear wheel. By interchanging the front and rear types, what is the greatest distance, in km, that can be travelled using a set of four tyres of this brand?

28. (SMOPS 13Q28) In triangle ABC, AD and AE trisect $\angle CAB$, BD and BE trisect $\angle CBA$. If the ratio of angle C to angle D is 1:2, find the value of angle E in degrees.



29. (SMOPS 13Q29) The sum of 10 positive integers, not necesarily distinct is 1001. If d is the greatest common divisor of the 10 numbers, find the maximum possible value of d.

30. (SMOPS 13Q30) How many different ways are there to select 2 distinct integers from $\{2000, 2001, 2002, \ldots, 2014, 2015\}$ such that the product of the 2 numbers is divisible by 6?

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