Mathlete Training Centre
SMOPS 2010

1. (SMOPS 10Q1) Find the value of $(1-\frac{1}{2})(1-\frac{1}{3})(1-\frac{1}{4})\cdots(1-\frac{1}{99})(1-\frac{1}{100})\times 1000$.

MATHLETE TRAINING CENTRE
PERSEVERANCE RIGOR DEDICATION 224 BISHAN STREET 23 BI-131

2. (SMOPS 10Q2) Find the value of the numerator of the sum $\frac{1}{2}+\frac{1}{2\times 2}+\frac{1}{2\times 2\times 2}+\cdots+\underbrace{\frac{1}{2\times 2\times\cdots\times 2}}_{10}$.

MATHLETE TRAINING CENTRE
PERSEVERANCE RIGOR DEDICATION 224 BISHAN STREET 23 BI-131

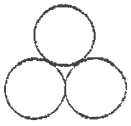
3. (SMOPS 10Q3) Find the last digit of $\underbrace{2 \times 2 \times 2 \times \cdots \times 2}_{859433}$.

4. (SMOPS 10Q4) Find the total number of ways to arrange 3 identical white balls and 3 identical black balls in a circle on a plane. The two layouts below are considered as one way of arrangment.

5. (SMOPS 10Q5) Given that one and only one of the following statements is correct, which one is correct?
   (1) All of the statements below are correct.
   (2) None of the statement below is correct.
   (3) One of the statements above is correct.
   (4) All the statements above are correct.
   (5) None of the statements above is correct.

MATHLETE TRAINING CENTRE
PERSEVERANCE RIGOR DEDICATION 224 BISHAN STREET 23 BI-131

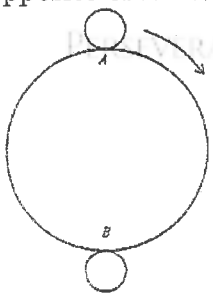
6. (SMOPS 10Q6) Three identical circles have at most three points of contact as shown below. Find the least number of identical circles required to have nine points of contact.



MATHLETE TRAINING CENTRE
PERSEVERANCE RIGOR DEDICATION 224 BISHAN STREET 23 BI-131

7. (SMOPS 10Q7) Find the exact value of $1^3 + 2^3 + 3^3 + \cdots + 20^3$.

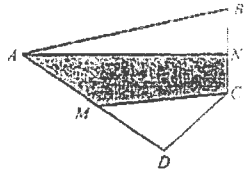
8. (SMOPS 10Q8) In the figure below, a circular disc of radius 1 rotates in a circular track of radius 6. Suppose the disc makes $z°$ turn about its center to reach the point B which is directly opposite of A. What is the value of z?



9. (SMOPS 10Q9) Let n be a whole number greater than 1. It leaves a remainder of 1 when divided by any single digit whole number greater than 1. Find the smallest possible value of n.

MATHLETE TRAINING CENTRE

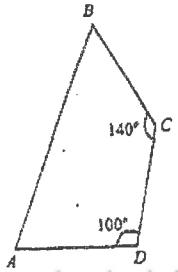
10. (SMOPS 10Q10) M and N are midpoints of the lines AD and BC respectively. Given that the area of $ABCD$ is 2000 cm^2 and the area of the shaded region $ANCM = x$ cm^2, find the value of x.



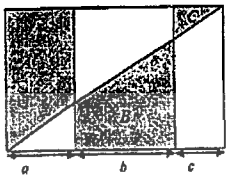
MATHLETE TRAINING CENTRE
PERSEVERANCE RIGOR DEDICATION 224 BISHAN STREET 23 BI-131

11. (SMOPS 10Q11) Your pocket money had previously been decreased by $x\%$. To get back to the same amount of pocket money before the decrease, you need to have an increase of 25%. Find the value of x.

12. (SMOPS 10Q12) In the diagram, $AD = DC = CB$ and $\angle ADC = 100°$ and $\angle DCB = 140°$, find $\angle ABC$.



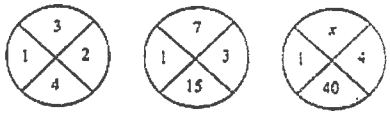
13. (SMOPS 10Q13) In the diagram below, the area of shaded region A and C is 8 cm^2 and 2 cm^2 respectively.



Given that the sum of a and c is equal to b, find the area of the shaded region B.

MATHLETE TRAINING CENTRE
PERSEVERANCE RIGOR DEDICATION 224 BISHAN STREET 23 BI-131

14. (SMOPS 10Q14) What is the number x?



MATHLETE TRAINING CENTRE
PERSEVERANCE RIGOR DEDICATION 224 BISHAN STREET 23 BI-131

15. (SMOPS 10Q15) When Albert begins walking up slope AB (1 km distance), across level ground BC (12 km distance), and down slope CD (3 km distance), Daniel begins his journey in the opposite direction from D at the same time. Given that the speeds of both travelling up the slope, on level ground and down slope are 2 km/h, 4 km/h and 5 km/h respectively, find the number of hours that has passed when they meet.

MATHLETE TRAINING CENTRE
PERSEVERANCE RIGOR DEDICATION 224 BISHAN STREET 23 BI-131

16. (SMOPS 10Q16) Three different bus services leave the bus terminal in every 24 minutes, 30 minutes and 36 minutes. At 0900 hour, the three bus services left the bus terminal. After how many hours did the three services again leave the terminal at the same time?

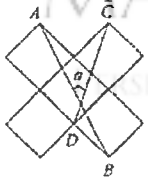
MATHLETE TRAINING CENTRE
PERSEVERANCE RIGOR DEDICATION 224 BISHAN STREET 23 BI-131

17. (SMOPS 10Q17) Twenty five boys position themselves in a 5 by 5 formation such that the distances between two adjacent boys in the same row or column are equal to 1 m. The two dark circles indicate a pair of boys whose distance apart is exactly 5 m. Given that there are n pairs whose distances apart are exactly 5 m, find the value of n.

MATHLETE TRAINING CENTRE
PERSEVERANCE RIGOR DEDICATION 224 BISHAN STREET 23 BI-131

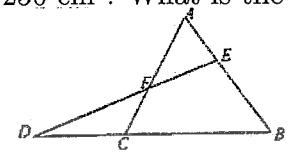
18. (SMOPS 10Q18) The figure below is formed by 5 identical squares. A, B, C and D are vertices of the squares and AB and CD are straight lines. Find the measure of angle a.

MATHLETE TRAINING CENTRE



PERSEVERANCE RIGOR DEDICATION 224 BISHAN STREET 23 BI-131

19. (SMOPS 10Q19) In the figure below, $AF = 2FC, EB = 2AE$. The area of triangle DEC is 250 cm^2. What is the area of $BCFE$?



MATHLETE TRAINING CENTRE
PERSEVERANCE RIGOR DEDICATION 224 BISHAN STREET 23 BI-131

20. (SMOPS 10Q20) P, Q, R, S and T are equally spaced on a straight rod. If the rod is first rotated 180° about T, then 180° about S and finally 180° about P, which point's position remains unchanged?

MATHLETE TRAINING CENTRE
PERSEVERANCE RIGOR DEDICATION 224 BISHAN STREET 23 BI-131

21. (SMOPS 10Q21) A goat on a horizontal ground is tied to one end of a 14 m long rope. The other red of the rope is attached to a ring which is free to slide along a fixed 20 m lng horizontal rail. If the maxmimum possible area that the goat can graze is x m^2, find the value of x. (Ignore the dimension of the ring and take π to be $\frac{22}{7}$.)

22. (SMOPS 10Q22) The 13 squares are to be filled with whole numbers. If the sum of any three adjacent numbers is 21, find the value of x.

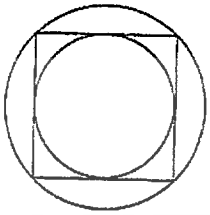
23. (SMOPS 10Q23) Given that $S = \dfrac{1}{\frac{1}{2001} + \frac{1}{2002} + \frac{1}{2003} + \cdots + \frac{1}{2009} + \frac{1}{2010}}$, find the largest whole number smaller than S.

MATHLETE TRAINING CENTRE

24. (SMOPS 10Q24) Find the smallest whole number that is not a factor of $1 \times 2 \times 3 \times \cdots \times 22 \times 23$.

MATHLETE TRAINING CENTRE
PERSEVERANCE RIGOR DEDICATION 224 BISHAN STREET 23 BI-131

25. (SMOPS 10Q25) A square has four vertices touching a circle and its four sides touching another smaller circle as shown below. If the area of the larger circle is x times that of the smaller one, find the value of x.

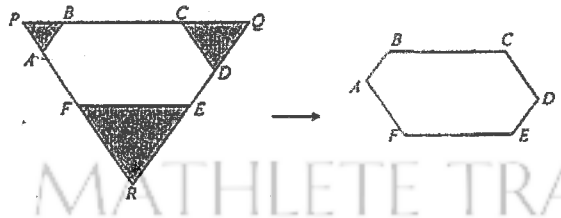


MATHLETE TRAINING CENTRE
PERSEVERANCE RIGOR DEDICATION 224 BISHAN STREET 23 BI-131

26. (SMOPS 10Q26) Given that the product of four different whole numbers is 10000, find the greatest possible value of the sum of the four numbers.

MATHLETE TRAINING CENTRE
PERSEVERANCE RIGOR DEDICATION 224 BISHAN STREET 23 BI-131

27. (SMOPS 10Q27) An equilateral triangle PQR of side 32 cm has three equilateral triangles cut off from its corners to give rise to a hexagon $ABCDEF$. Another equilateral triangle of side x cm gives rise to the same hexagon when subjected to the same treatment. If $AB = 8$ cm, $BC = 15$ cm, $CD = 9$ cm, $DE = 10$ cm, $EF = 13$ cm and $FA = 11$ cm, find the value of x.



MATHLETE TRAINING CENTRE
PERSEVERANCE RIGOR DEDICATION 224 BISHAN STREET 23 BI-131

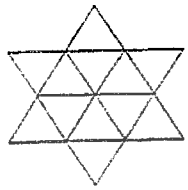
28. (SMOPS 10Q28) Given the following three numbers A, B and C:

$$A = \underbrace{3 \times 3 \times 3 \times \cdots 3}_{40} \qquad B = \underbrace{5 \times 5 \times 5 \times \cdots 5}_{30} \qquad C = \underbrace{7 \times 7 \times 7 \times \cdots 7}_{20}$$

MATHLETE TRAINING CENTRE
PERSEVERANCE RIGOR DEDICATION 224 BISHAN STREET 23 BI-131

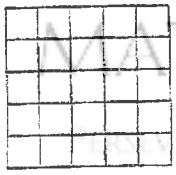
The ordr of these three numbers from largest to smallest is _____

29. (SMOPS 10Q29) The figure comprises twelve equilateral triangles. Find the total number of trapeziums in the figure. Here we define a trapezium to be a 4-sided figure with exactly one pair of parallel sides.



MATHLETE TRAINING CENTRE
PERSEVERANCE RIGOR DEDICATION 224 BISHAN STREET 23 BI-131

30. (SMOPS 10Q30) The following 5 by 5 grid consists of 25 unit squares. Find the largest number of unit squares to be shaded so that each row, each column and each of the two main diagonals has at most 2 unit squares that are shaded.



MATHLETE TRAINING CENTRE

Perseverance Rigor Dedication 224 Bishan Street 23 B1-131

MATHLETE TRAINING CENTRE

Perseverance Rigor Dedication 224 Bishan Street 23 B1-131

MATHLETE TRAINING CENTRE

Perseverance Rigor Dedication 224 Bishan Street 23 B1-131