Mathlete Training Centre
SMOPS 2009

1. (SMOPS 09Q1) Given that the product of two whole numbers $m \times n$ is a prime number, and the value of $m$ is smaller than $n$, find the value of $m$.



2. (SMOPS 09Q2) Given that $(2009 \times n - 2009) \div (2008 \times 2009 - 2006 \times 2007) = 0$, find the value of $n$.



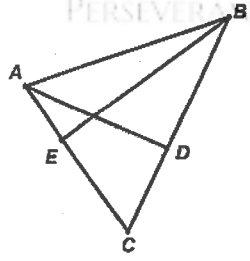


3. (SMOPS 09Q3) Find the missing number $x$ in the following sequence.
$2, 9, -18, -11, x, 29, -58, -51, \ldots$

4. (SMOPS 09Q4) Jane has 9 boxes with 9 accompanying keys. Each box can only be opened by its accompanying key. If the 9 keys have been mixed up, find the maximum number of attempts Jane must make before she can open all the boxes.



5. (SMOPS 09Q5) The diagram shows a traingle $ABC$ with $AC = 18$ cm and $BC = 24$ cm. $D$ lies on $BC$ such that $AD$ is perpendicular to $BC$. $E$ lies on $AC$ such that $BE$ is perpendicular to $AC$. Given that $BE = 20$ cm and $AD = x$ cm, find the value of $x$.



6. (SMOPS 09Q6) A language school has 100 pupils in which 69% of the pupils study French, 79% study German, 89% study Japanese and 99% study English. Given that at least $x$% of the students study all four languages, find the value of $x$.



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7. (SMOPS 09Q7) Find the value of $x$.

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| | | | |
|---|---|---|---|
| 89 | 35 | 9 | 1 |
| 35 | $x$ | 7 | 1 |
| 9 | 7 | 5 | 1 |
| 1 | 1 | 1 | 3 |

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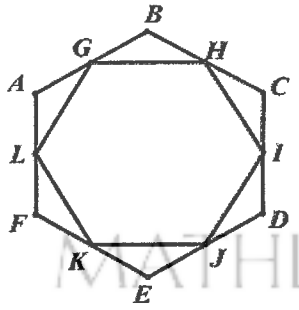
8. (SMOPS 09Q8) Given that $9 = n_1 + n_2 + n_3 + n_4 + n_5 + n_6 + n_7 + n_7 + n_8 + n_9$ where $n_1, n_2, n_3, n_4, n_5, n_6, n_7, n_8, n_9$ are consecutive whole numbers, find the value of the product $n_1 \times n_2 \times n_3 \times n_4 \times n_5 \times n_6 \times n_7 \times n_8 \times n_9$.





9. (SMOPS 09Q9) The diagram shows a regular 6-sided figure $ABCDEF$. $G, H, I, J, K, L$ are midpoints of $AB, BC < CD < DE < EF$ and $FA$ respectively. Given that the area of $ABCDEF$ is 100 cm$^2$ and the area of $GHIJKL$ is $x$ cm$^2$, find the value of $x$.



10. (SMOPS 09Q10) Three pupils $A, B$ and $C$ are asked to write down the height of a child, the circumference of a circle, the volume of a cup and the weight of a ball. Their responses are tabulated below: If each pupil has only two correct responses and the height of the child is $x$ cm, find the value of $x$.

| Pupil | Height of the child (cm) | Circumference of the circle (cm) | Volume of the cup ($cm^3$) | Weight of the ball (g) |
|---|---|---|---|---|
| *A* | 90 | 22 | 250 | 510 |
| *B* | 70 | 21 | 245 | 510 |
| *C* | 80 | 22 | 250 | 520 |



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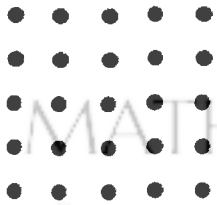
11. (SMOPS 09Q11) The diagram shows a square grid comprising 25 dots. A circle is attached to the grid. Find the largest possible number of dots the circle can pass through.



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12. (SMOPS 09Q12) Jane and Peter competed in a 100 m race. When Peter crossed the finishing line, Jane just crossed the 90 m mark. If Peter were to start 10 m behind the starting line, the distance between them when one of them crosses the finishing line is $x$ m, Find the value of $x$.



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13. (SMOPS 09Q13) Given that $(1+2+3+4+5+4+3+2+1) \times (123454321) = x^2$, find the value of $x$.

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14. (SMOPS 09Q14) A $5 \times 5 \times 5$ cube is to be assembled using only $1 \times 1 \times 1$ cuboid(s) and $1 \times 1 \times 2$ cuboid(s). Find the maximum number of $1 \times 1 \times 2$ cuboid(s) required to build this $5 \times 5 \times 5$ cube.





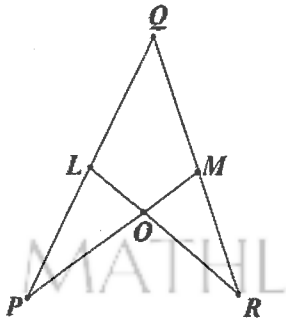
15. (SMOPS 09Q15) Given that $(n_1)^2 + (2n_2)^2 + (3n_3)^2 + (4n_4)^2 + (5n_5)^2 + (6n_6)^2 + (7n_7)^2 + (8n_8)^2 + (9n_9)^2 = 285$, find the value of $n_1 + n_2 + n_3 + n_4 + n_5 + n_6 + n_7 + n_8 + n_9$ if $n_1, n_2, n_3, n_4, n_5, n_6, n_7, n_8, n_9$ are non-zero whole numbers.

16. (SMOPS 09Q16) A circle and a square have the same perimeter. Which of the following statements is true?
    (1) Their areas are the same.
    (2) The area of the circle is four times the area of the square.
    (3) The area of the circle is greater than that of the square.
    (4) The area of the cirle is smaller than that of the square.
    (5) None of the above.





17. (SMOPS 09Q17) As shown in the diagram, the points $L$ and $M$ lie on $PQ$ and $QR$ respectively. $O$ is the point of intersection of the lines $LR$ and $PM$. Given that $MP = MQ, LQ = LR, PL = PO$ and $\angle POR = x°$, find the value of $x$.



18. (SMOPS 09Q18) Given that the value of the sum $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}$ lies between $\frac{28}{29}$ and 1, find the smallest possible value of $a+b+c$ where $a, b$ and $c$ are whole numbers.



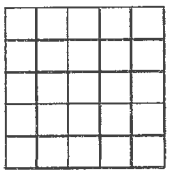
19. (SMOPS 09Q19)Jane has nine 1 cm long sticks, six 2 cm long sticks and three 4 cm long sticks. Given that Jane has to use all the sticks to make a single rectangle, how many rectangles with different dimensions can she make?

20. (SMOPS 09Q20) Peter wants to cut a 63 cm long string into smaller segments so that one or more of the segments add up to whole numbers in centimeters from 1 to 63. Find the least number of cuts he must make.





21. (SMOPS 09Q21) The diagram shows a 5 by 5 square comprising twenty five unit squares. Find the least number of unit squares to be shaded so that any 3 by 3 square has exactly four unit squares shaded.



22. (SMOPS 09Q22) Peter and Jane were each given a candle. Jane's candle was 3 cm shorter than Peter's and each candle burned at a different rate. Peter and Jane lit their candles at 7 pm and 9 pm respectively. Both candles burned down to the same height at 10 pm. Jane's candle burned out after another 4 hours and Peter's candle burned out after another 6 hours. Given that the height of Peter's candle at the beginning was $x$ cm, find the value of $x$.



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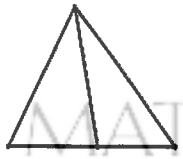
23. (SMOPS 09Q23) Three straight lines can form a maximum of one triangle. Four straight lines can form a maximum of two non-overlapping triangles as shown below. Five straight lines can form a maximum of five non-overlapping triangles. Six straight lines can form a maximum of $x$ non-overlapping triangles. Find the value of $x$.



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24. (SMOPS 09Q24) Given that $N = \underbrace{2 \times 2 \times 2 \times \cdots \times 2}_{2009} \times \underbrace{5 \times 5 \times 5 \times \cdots \times 5}_{2000}$, find the number of digits in $N$.



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25. (SMOPS 09Q25) Jane and Peter are queueing uo in a single line to buy food at the canteen. There are $x$ persons behind Jane and there are $y$ persons in front of Peter. Jane is $z$ persons in front of Peter. The number of people in the queue is _____ persons.
(1) $-x+y+z-1$
(2) $x+y-z+1$
(3) $-x+y+z$
(4) $-x+y+z+2$
(5) $x+y-z$

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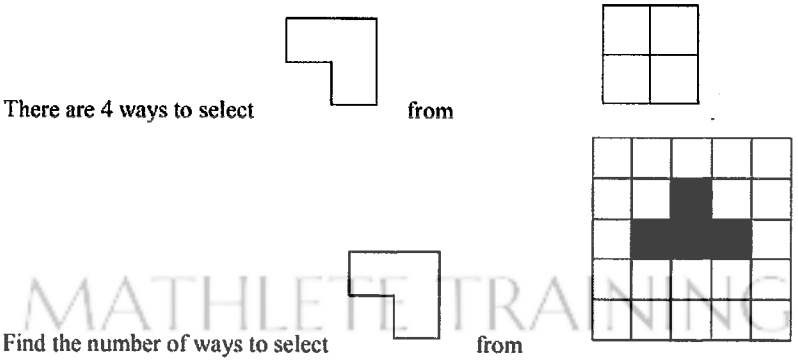
26. (SMOPS 09Q26)



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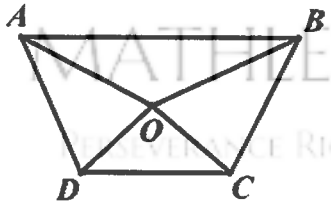
27. (SMOPS 09Q27) The diagram shows a trapezium $ABCD$. The length of $AB$ is $2\frac{1}{2}$ times that of $CD$ and the areas of triangles $OAB$ and $OCD$ are 20 cm$^2$ and 14 cm$^2$ respectively. Given that the area of the trapezium is $x$ cm$^2$, find the value of $x$.

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28. (SMOPS 09Q28) Given that $\frac{1}{10+\frac{1}{9+\frac{1}{9}}} + \frac{1}{a+\frac{1}{b+\frac{1}{b}}} = 1$, where $a$ and $b$ are whole numbers, find the value of $a + b$.



29. (SMOPS 09Q29) A shop sells dark and white chocolates in three different types of packaging as shown in the table. Mr Tan bought a total of 36 packages which consisted of 288 pieces of dark chocolates and 105 pieces of white chocolates. How many packages of type A did he buy?

| | *Number of Dark Chocolate* | *Number of White Chocolate* |
|---|---|---|
| Package *A* | 9 | 3 |
| Package *B* | 9 | 6 |
| Package *C* | 6 | 0 |

30. (SMOPS 09Q30) There are buses travelling to and fro between Station A and Station B. The buses leave the stations at regular intervals and a bus will meet another bus coming in the opposite direction every 6 minutes. Peter starts cycling from A towards B at the same time Jane starts cycling from B to A. Peter and Jane will meet a bus coming in the opposite direction every 7 and 8 minutes respectively. After 56 minutes of cycling on the road, they meet each other. Find the time taken by a bus to travel from A to B.

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