Mathlete Training Centre
SMOPS 2008

1. (SMOPS 08Q1) What is the difference between the sum of the first 2008 even numbers and the sum of the first 2008 odd numbers?

2. (SMOPS 08Q2) The sides of a triangle have lengths that are consecutive whole numbers and its perimeter is greater than 2008 cm. If the least possible perimeter of the triangle is x cm, find the value of x.

3. (SMOPS 08Q3) Find the value of $2008 \times 20072007 - 2007 \times 20082007$.

4. (SMOPS 08Q4) When a rectangular sheet of paper with length 8 cm is folded exactly into half, the ratio of its length to its width remains unchanged. If the square of the width of the original piece of paper $= x$ cm^2, find the value of x.

5. (SMOPS 08Q5) The numbers 1 to 10 are arranged in the circles in such a way that the sum of the four numbers on each line is 21. What is the value of n?
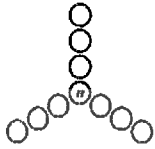
6. (SMOPS 08Q6) Find the value of $(56789 + 67895 + 78956 + 89567 + 95678) \div 5$.

7. (SMOPS 08Q7) The diagram shows a circle whose circumference touches the sides and the vertices of a large and a small square respectively. If the area of the small square is 9 cm^2 and the area of the large square is x cm^2, find the value of x.

8. (SMOPS 08Q8) 8 whole numbers are placed along the circumference of a circle. When any five adjacent numbers are added, the total is always 40. Find the largest possible difference between the largest and the smallest of these numbers.

9. (SMOPS 08Q9) In triangle PQR, $PQ = 6$ cm, $PR = 4$ cm and $QR = 6$ cm. If sides PQ and PR are tripled while QR remains unchanged, then
   (1) the area is tripled.
   (2) the area increases by 9 times.
   (3) the altitude is tripled.
   (4) the area decreases to 0 cm^2.
   (5) None of the above.

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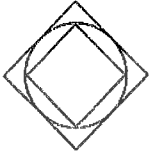PERSEVERANCE RIGOR DEDICATION 224 BISHAN STREET 23 B1-131

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10. (SMOPS 08Q10) Find the last 5 digits of the sum
    $1 + 22 + 333 + 4444 + 55555 + 666666 + 7777777 + 88888888 + 999999999$.

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11. (SMOPS 08Q11) If an arc of 80° on circle A has the same length as an arc of 60° on circle B, and that the ratio of area of circle A to the area of circle B is $a : b$, find the smallest possible value of $a + b$.

12. (SMOPS 08Q12) A circle of circumference 1 m rolls around the equilateral triangle of perimeter 3 m. How many turns does the circle make as it rolls around the triangle once without slipping?

13. (SMOPS 08Q13) The diagram shows a rectangle $ABCD$ with area 32 cm^2. Given that area of triangle ADF= 2 cm^2, area of triangle ABE= 8 cm^2 and area of the shaded region = x cm^2, find the value of x.
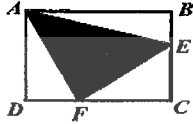
14. (SMOPS 08Q14) There are two containers A and B. Each of them contains 9 white marbles, 9 black marbles and 9 red marbles. If 10 marbles are removed from A and placed into B, how many marbles must be returned from B to A to make sure that there are at leat 8 marbles of each colour in A?

15. (SMOPS 08Q15) 9^{10} is a 10-digit number. If A is the sum of all digits of 9^{10}, B is the sum of all digits of A and C is the sum of all digits of B, find the value of C.

16. (SMOPS 08Q16) A car travels from point A to B at constant speed of V km/h. If the car increases its speed by 20%, it will reach B one hour earlier. If the car increases its speed by 25% after travelling at V km/h for 120km, it will reach B forty eight minutes earlier. If the distance between the two towns is x km, find the value of x.

17. (SMOPS 08Q17) After all the faces of a rectangular block are painted green, the block is cut into unit cubes of volume 1 cm^3. It is found that 7 of the unit cubes have none of their faces painted green. How many of the unit cubes have exactly two faces painted green?

18. (SMOPS 08Q18) The diagram shows an equilateral triangle ABC with OD, OE and OF perpendicular to AB, BC and CA respectively. If $OD + OE + OF = 28$ cm and the height of triangle $ABC = x$ cm, find the value of x.
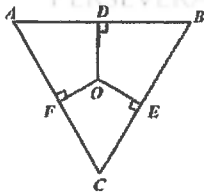
19. (SMOPS 08Q19) The diagram shows a triangle POR. Three lines parallel to the sides of the triangle are drawn through a point O. Given that the areas of the three shaded parallelograms are 32 cm^2, 48 cm^2 and 96 cm^2 respectively, and the area of the triangle $PQR = x$ cm^2, find the value of x.
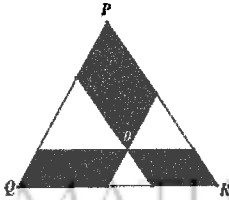

20. (SMOPS 08Q20) As shown in the diagram, a straight line can cut across at most 3 squares in a 2 by 2 square and at most 5 squares in a 3 by 3 square. What is the greatest number of squares that can be cut across by a straight line in a 2008 by 2008 square?

21. (SMOPS 08Q21) There are three containers. One contains red marbles, another white marbles and the third one a mixture of red and white marbles. Given that all of them are labelled wrongly and you are allowed to open only one of them to take out only one marble in order to state correctly where all the labels ought to go, which container should you open?
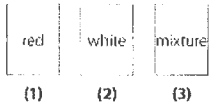
red	white	mixture
(1)	(2)	(3)

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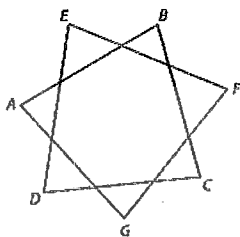PERSEVERANCE RIGOR DEDICATION 224 BISHAN STREET 23 B1-131

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22. (SMOPS 08Q22) Given that $\underbrace{2000820082008\ldots2008623}_{n \text{ of } 2008}$, find the smallest value of n such that the number is divisible by 11.

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23. (SMOPS 08Q23) Given that $\angle ABC + \angle BCD + \angle CDE + \angle DEF \angle EFG + \angle FGA + \angle GAB = x°$, find the value of x.

24. (SMOPS 08Q24) Peter and Jane are to take turns to subtract perfect squares from a given whole number and the one who reaches zero first is the winner. If the whole number is 29 and Peter is the first player, what perfect square must he subtract in order for him to definitely win?

25. (SMOPS 08Q25) The inhabitants of an island are either gentleman or liars. A gentleman always tells the truth and a liar always lies. A, B and C are three of the inhabitants. A sailor who landed on the island asked A:"Are you a gentleman or a liar?" A answered but the sailor could not hear clearly what he said. He then asked B:"What did A say?" B replied: "A said that he is a liar". At that instant, C immediately shouted:"B is lying!"

I It is impossible to tell whether A is a gentleman or a liar.
II B is a gentleman and C is a liar
III B is a liar and C is a gentleman.

(1) Only I is true.
(2) Only II is true.
(3) Only III is true.
(4) Only I and II are true.
(5) Only I and III are true.

26. (SMOPS 08Q26) The product of n whole numbers $1 \times 2 \times 3 \times 4 \times 5 \times \ldots \times (n-1) \times n$ has twenty eight consecutive seros. Find the largest value of n.

27. (SMOPS 08Q27) Find the largest number n such that there is only one whole number k that satisfies $\frac{8}{21} < \frac{n}{n+k} < \frac{5}{13}$.

(Note: $A < C < B$ means that the value of C is between A and B, example $4 < 9 < 16$)

28. (SMOPS 08Q28) How many ways are there to distribute 28 identical marbles in 3 different boxes such that no box is empty?
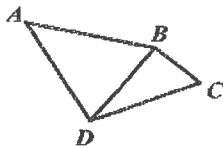
29. (SMOPS 08Q29) If Peter walks up an up-going escalator at the rate of 1 step per second, he is able to reach the top in 10 steps. If he increases his rate to 2 steps per second, he can reach the top in 16 steps. Find the number of steps of the escalator.

30. (SMOPS 08Q30) The diagram shows a quadrilateral $ABCD$. If $AB = CD$, $\angle ADB + \angle CBD = 180°$, $\angle BCD = 55°$ and $\angle BAD = x°$, find the value of x.

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