Mathlete Training Centre
SMOPS 2007

1. (SMOPS 07Q1) Find the value of $(1+\frac{2}{1}) \times (1+\frac{2}{2}) \times (1+\frac{2}{3}) \times (1+\frac{2}{4}) \times \cdots \times (1+\frac{2}{26}) \times (1+\frac{2}{27})$.
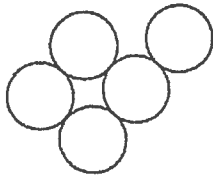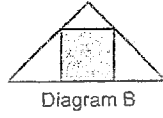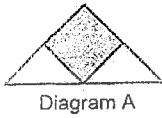
2. (SMOPS 07Q2) The diagram shows 5 identical circles. On the answer sheet provided, draw a straight line to divide the figure into two parts of equal area.

3. (SMOPS 07Q3) The diagram shows two identical isoceles right-angled triangles. If the area of the shaded square in diagram A is 50 cm^2, what is the area of the shaded square in diagram B?
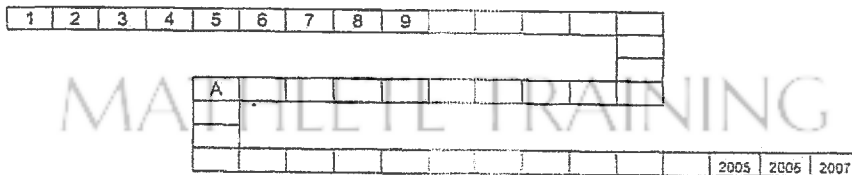

Diagram A   Diagram B

4. (SMOPS 07Q4) A rectangular wooden block measuring 30 cm by 10 cm by 6 cm is cut into as many cubes of side 5 cm as possible. Find the volume of the remaining wood.

5. (SMOPS 07Q5) The diagram shows 2007 identical retangles arranged as shown. What number does A represent?

6. (SMOPS 07Q6) The diagram shows a trapezium $ABCD$ with $AD = BC$. If $BD = 7$ cm, $\angle ABD = 45°$, find the area of the trapezium.
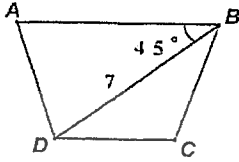
7. (SMOPS 07Q7) An organism reproduces by simple division into two. Each division takes 5 minutes to complete. When such an organism is placed in a container, the container is filled with organisms in 1 hour. How long would it take for the container to be filled if we start with two such organisms?
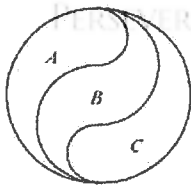
8. (SMOPS 07Q8) Given that a, b and c are different whole numbers from 1 to 9, find the largest possible value of $\frac{a+b+c}{a \times b \times c}$.

9. (SMOPS 07Q9) The diagram comprises a circle of radius 3 cm, two semicircles of radii 2 cm and two semicircles of radii 1 cm. Find the ratio of the areas of the regions A, B and C.

10. (SMOPS 07Q10) In 2005 both John and Mary have the same amount of pocket money per month. In 2006, John had an increase of 10% and Mary a decrease of 10% in their pocket money. In 2007, John had a decrease of 10% and Mary an increase of 10% in their pocket money. Which one of the following statements is correct?
(A) Both have the same amount of pocket money now.
(B) John has more pockey money now.
(C) Mary has more pocket money now.
(D) It is impossible to tell who has more pocket money now.

11. (SMOPS 07Q11) A set of 9-digit numbers each of which is formed by using each of the digits 1 to 9 once and only once. How many of these numbers are prime?

12. (SMOPS 07Q12) Water expands 10% when it freezes to ice. Find the depth of the water to which a regular container of base 22 cm by 33 cm and height 44cm is to be filled so that when the water freezes completely to ice it will fill the container exactly.

13. (SMOPS 07Q13) The diagram shows two circles with centre O. Given that line AB is a chord 14 cm long and just touches the circumference of the shaded circle, find the area of the non-shaded region. Take π as $\frac{22}{7}$.
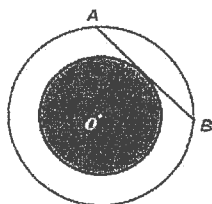
14. (SMOPS 07Q14) Joan could cycle 1 km in 4 minutes with the wind and returned in 5 minutes against the wind. How long would it take her to cycle 1 km if there is no wind? Assuming her cycling speed and the wind speed are constant throughout the journey.

15. (SMOPS 07Q15) Given that $\sqrt{1 + 1 \times 2 \times 3 \times 4} = 5$, $\sqrt{1 + 2 \times 3 \times 4 \times 5} = 11$, $\sqrt{1 + 3 \times 4 \times 5 \times 6} = 19$ and $\sqrt{1 + 4 \times 5 \times 6 \times 7} = 29$, find the value of $\sqrt{1 + 204 \times 205 \times 206 \times 207}$.

16. (SMOPS 07Q16) Peter walked once around a track and Jane ran several times around it in the same direction. They left the starting point at the same time and returned to it at the same time. In between Jane overtook Peter twice. If she had run around the track in the opposite direction how many times would she have passed Peter? Assume that their speeds had been constant throughout the journey.

17. (SMOPS 07Q17) Three clocks, with their hour hands missing, have minute hands which run faster than normal. Clocks A, B and C each gains 2, 6 and 15 minutes per hour respectively. They start at noon with all three minutes hands pointing to 12. How many hours have passed before all three minute hands next point at the same time?

18. (SMOPS 07Q18) $ABCD$ is a rectangle and AEF, BEH, HGC, FGD are straight lines. Given that the area of the 4-sided figure $EFGH$ is 82 cm^2, find the total area of the shaded regions.
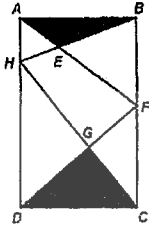
19. (SMOPS 07Q19) Four cards, each with a letter on one side and a number on the other side, are laid on a table. John claims that any card with a letter A on one side always has the number 1 on the other side. Which two of the four cards would you turn over to check his statement?
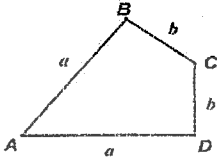
20. (SMOPS 07Q20) The diagram shows a 4-sided figure $ABCD$ with $AB = AD = a$ cm and $BC = CD = b$ cm where a is greater than b and that both a and b are whole numbers. Given that the area is 385 cm^2, find the smallest perimeter of the figure.

21. (SMOPS 07Q21) Find the number of ways to put 4 different coloured marbles into 4 identical empty boxes.

22. (SMOPS 07Q22) Abel and Bernard started travelling at the same instant from P and reached S at the same time. During the journey, Abel spent one third of Bernard's travelling time resting while Bernard spent one quarter of Abel's travelling time resting. Find the ratio of Abel's speed to Bernard's speed. (Note: travelling time excludes time for resting)

23. (SMOPS 07Q23) A family consists of 1 grandfather, 2 fathers, 1 father-in-law, 1 brother, 2 sons, 1 grandmother, 2 mothers, 1 mother-in-law, 1 daughter-in-law, 2 sisters, 2 daughters, 4 children and 3 grandchildren. What is the smallest possible number of persons in this family?
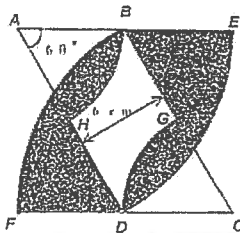
24. (SMOPS 07Q24) There are 5 schools between school A and school B. The seven schools are whole number of kilometers from each other along a straight line. The schools are spaced in such a way that if one knows the distance a person has travelled between any two schools he can identify the two schools. What is the shortest distance from A to B?
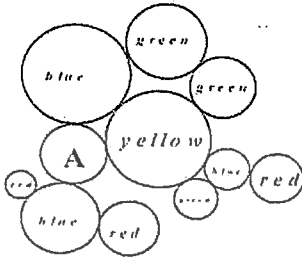
25. (SMOPS 07Q25) The diagram shows a parallelogram $ABCD$ with $\angle BAD = 60°$, $AB = 7$ cm, $AD = 14$ cm and a height of 6 cm. Arcs BH and ED have centres at A and arcs BF and GD have centres at C. Given that ABE, FDC, AHD, BGC are straight lines, find the total area of the shaded regions. Take π as $\frac{22}{7}$.

26. (SMOPS 07Q26) What is the colour of circle A?

27. (SMOPS 07Q27) Find the value of $(1-\frac{1}{2^2})(1-\frac{1}{3^2})(1-\frac{1}{4^2})(1-\frac{1}{5^2})\cdots(1-\frac{1}{2006^2})(1-\frac{1}{2007^2})$.

28. (SMOPS 07Q28) Three circles of radii 7cm are enclosed by a belt as shown in the diagram below. By rearranging the circles, it is possible to find the shortest possible lenggth of the belt needed to enclose the three circles. Find this shortest length. Take π as $\frac{22}{7}$.

29. (SMOPS 07Q29) Given that $\frac{1}{n_1} > \frac{2}{n_2} > \frac{3}{n_3} > \cdots > \frac{99}{n_{99}} > \frac{100}{n_{100}}$ and $n_1, n_2, n_3, \ldots, n_{99}, n_{100}$ are different whole numbers, find the smallest value of the sum $n_1 + n_2 + n_3 + \cdots + n_{99} + n_{100}$.

30. (SMOPS 07Q30) A ball was dropped from a height of 270 m, On each rebound, it rose to 10% of the previous height. Find the total vertical distance travelled by the ball before coming to rest.
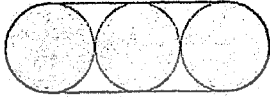
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