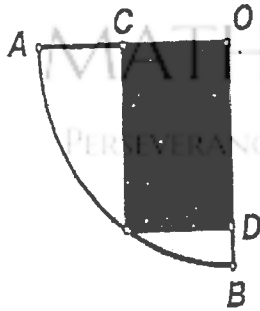Mathlete Training Centre
SMOPS 2006

1. (SMOPS 06Q1) The diagram shows a quarter circle OAB centre O with radius 6 cm. The shaded region is a rectangle. Find the length CD.

2. (SMOPS 06Q2) Draw straight lines to divide the figure into 4 identical shapes with equal areas.

3. (SMOPS 06Q3) A wooden plank rests on four identical rollers on horizontal ground. The circumference of each roller is 1 m, Find the distance moved by the plank after the rollers have each made one complete revolution, assuming there is no slipping.

4. (SMOPS 06Q4) Both Jane and Mary had a sum of money each. They wanted to buy a Mathematics book. They were short of $52 and $2 respectively and after pooling their money together, they still could not afford the book. Given that the price of the book is a whole number, how much did the book cost?

5. (SMOPS 06Q5) In a social gathering, each of the 38 boys talked to 3 girls while each girl talked to 2 boys. Find the number of girls in the social gathering.

6. (SMOPS 06Q6) What is the missing number in the following number sequence?
4, 6, 10, 14, 22, 26, _____, 38, 46, 58

7. (SMOPS 06Q7) In a party, there is a group of children. The boys always lie and the girls never lie. 10 children make the following statements.

A: Only one of us is a boy.
B: Only two of us are boys.
C: Only three of us are boys.
D: Only four of us are boys.
E: Only five of us are boys.
F: Only six of us are boys.
G: Only seven of us are boys.
H: Only eight of us are boys.
I: Only nine of us are boys.
J: All of us are boys.

Which of them is/are girl(s)?

8. (SMOPS 06Q8) Every morning Kevin has to reach school by 8 am.
If he travels at 70 metres per minute, he would arrive 10 minutes earlier.
If he travels at 60 metres per minute, he would arrive 8 minutes earlier.
Given that he leaves home for school after 7 a.m., find the exact time he leaves for school.

9. (SMOPS 06Q9) The diagram shows 12 identical rectangles within a larger rectangle with one side 54 cm long. Find the total area of the shaded regions.
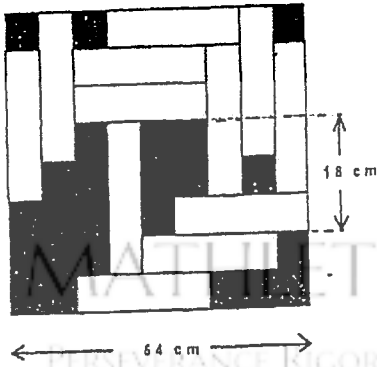
10. (SMOPS 06Q10) The prices of 3 types of snacks, a chocolate bar, a potato pie and a piece of cake were $1.50, $2.10, $3.30 respectively. Ben had $14 to spend on these snacks and he bought the snacks in such a way that he maximised his spending. What was the amount he had left?

11. (SMOPS 06Q11) In the triangle ABC, $\angle ABC = 76°$, $AD = AF$ and $CE = CF$. Find $\angle DFE$.
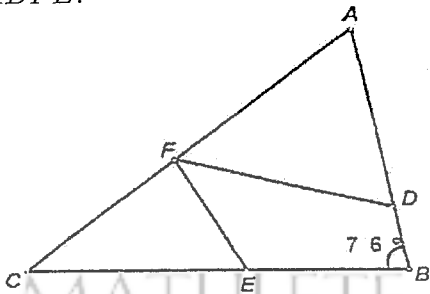
12. (SMOPS 06Q12) Two running tracks A (inner dotted track) and B (outer solid track) are each formed from straight parallel sides and semicircular ends. The tracks are 1 m apart. How much longer is track B compared to track A? Take π as $\frac{22}{7}$.
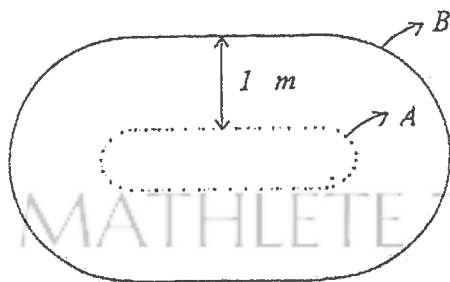
13. (SMOPS 06Q13) It takes 15 hours to fill up a tank when both taps A and B are turned on together. If tap A is turned on for 8 hours, then turned off, tap B will take another 50 hours to fill up the tank. How long will it take for the tank to be filled up by tap B alone?

14. (SMOPS 06Q14) The diagram shows a square $ABCD$ formed by 9 identical squares. Vertices $A, E, F, G, H, I, J, K, L$ and M are each joined to vertex C. What is the sum of the angles a, b, c, d, e, f, g, h and i?
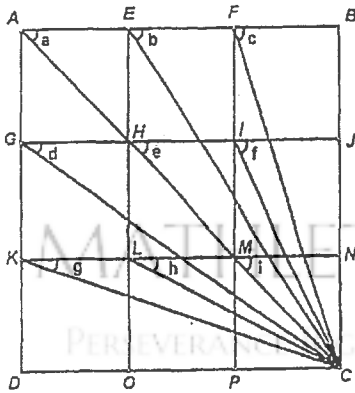
15. (SMOPS 06Q15) There is a row of coins numbered from 1 to 2006, each with head facing up.
In the first round, coins whose numbers are divisible by 1 are turned over.
In the 2nd round, coins whose numbers are divisible by 2 are turned over.
In the 3rd round, ...
In the 2006th round, coins whose numbers are divisible by 2006 are turned over. How many coins will have head facing up after the 2006th round of turning?

16. (SMOPS 06Q16) The diagram shows a figure consisting of three identical squares of side 1 m. 10 points A, B, CD, E, F, G, H, I and J and the vertices of the squares. How many triangles, of area 1 m^2, can be formed using any 3 of the 10 points as vertices, given that the triangle must have one of its sides either horizontal or vertical?
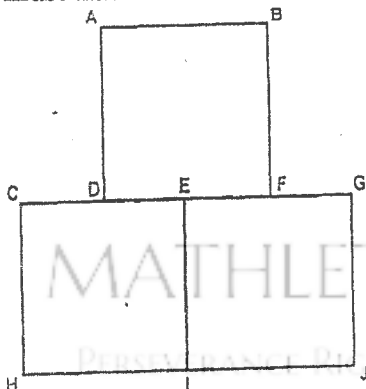
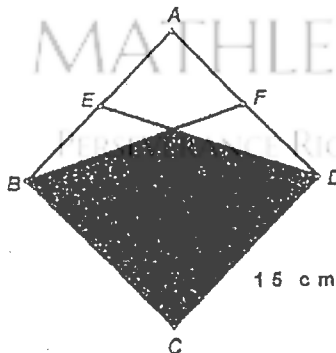17. (SMOPS 06Q17) Find the value of $\left(\frac{1}{2} + \frac{345}{456} + \frac{567}{678} + 1\right) \times \left(\frac{345}{456} + \frac{567}{678} + \frac{7}{8}\right) - \left(\frac{1}{2} + \frac{345}{456} + \frac{567}{678}\right) \times \left(\frac{345}{456} + \frac{567}{678} + \frac{7}{8} + 1\right)$

18. (SMOPS 06Q18) $ABCD$ is a square of side 15 cm. E and F are the midpoints of AB and AD respectively. Find the area of the shaded region.

19. (SMOPS 06Q19) There are 3 boxes. One box contains a white ball, one a black ball and one a doll.

A sentence is written on the cover lid of each of the boxes.

The sentence written on the box containing the white ball is always true.

The sentence written on the box containing the black ball is always false.

The sentence written on the box containing the doll can be true or false.

Box 1: The sentence written on Box 2 is true.

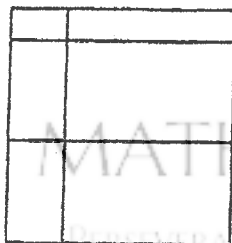Box 2: Box 1 contains a black ball.

Box 3: The doll is in Box 1.

Which box contains the doll?

20. (SMOPS 06Q20) The diagram shows a square formed by 6 rectangles. Given that the total perimeter of the 6 rectangles is 220 cm, find the area of the square.

21. (SMOPS 06Q21) There are 2006 oranges.
   The first group of people consumed $\frac{1}{2}$ of the oranges,
    the 2nd group of people consumed $\frac{1}{3}$ of the remaining oranges,
    the 3rd group ...,
    and the 2005th group of people consumed $\frac{1}{2006}$ of the remaining oranges.
    Find the number of oranges left.

22. (SMOPS 06Q22) The diagram shows a semicircle with diameter 14cm. PQS and QPR are
    two quater circles with P and Q as centres respectively. Find the difference between the areas
    of the shaded region A and region B. Take π as $\frac{22}{7}$.
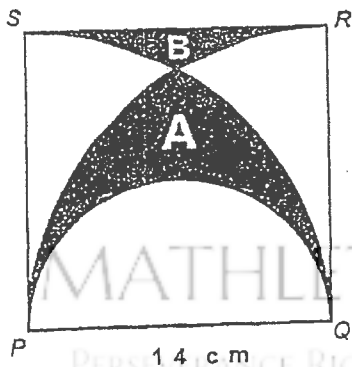
23. (SMOPS 06Q23) Ling Ling started from Town A and planned to reach Town B by noon. For the first half of her journey, her speed was $\frac{21}{23}$ of the planned speed. If she arrived in town B at noon, she must have travelled at _____ of the planned speed in the second half of the journey.

24. (SMOPS 06Q24) The diagram shows a beam balance. By placing strandard weights and the object to be measured at the ends of the beam, it can measure the weight of an object. What is the minimum number of standard weights required to weigh any object from 1 to 40 kg, given that the standard weights and the object can only have whole number weights?
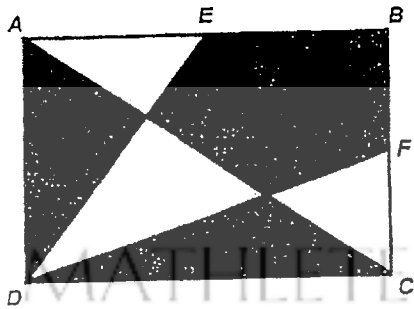
[Note : The diagram below shows that the weight of the object is 2 kg. 2 kg is a standard weight.]

25. (SMOPS 06Q25) $ABCD$ is a rectangle with area 150 cm^2. E and F are the midpoints of AB and BC respectively. Find the total area of the shaded parts.

26. (SMOPS 06Q26) Find the value of $1 + \frac{1}{3} + \frac{1}{6} + \frac{1}{10} + \frac{1}{15} + \frac{1}{21} + \cdots + \frac{1}{300}$.

27. (SMOPS 06Q27) How many 3-digit numbers have a remainder of 7,2 and 3 when divided by 9, 5 and 4 respectively?

28. (SMOPS 06Q28) Samuel wants to distribute 174 identical marbles into a number of boxes. Given that each box can have 12 to 22 marbles and that each box contains a different number of marbles, find the number of different ways to do the task.

29. (SMOPS 06Q29) Alan and Ben start running, at the same instant, along the running track from A and B respectively. They are running way from each other until they meet each other at C. After which, Ben turns to run in the same direction as Alan. When Ben passes B, Alan passes A. Given that Alan has run 800 m in total when he meets Ben the second time and that $AB = BC$, find the total length of the running track. Assume that the speeds have been constant throughout the journey.
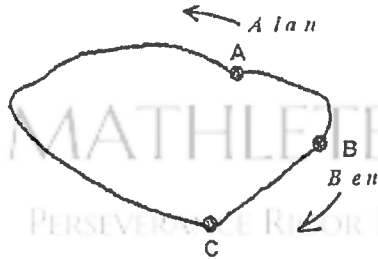
30. (SMOPS 06Q30) Take a number between 100 and 999. The digits of the number are then rearranged so that no digit occupies its original position. The difference between these two numbers is less than 100 and is perfect cube. List all such possible numbers.

MATHLETE TRAINING CENTRE

Perseverance Rigor Dedication 224 Bishan Street 23 BI-131