

Mathlete Training Centre
RIPMWC 2021

1. If $A = 9 - 99 + 999 - 9999 + \dots - \underbrace{99\dots99}_{2020\text{'9s'}} - \underbrace{99\dots99}_{2021\text{'9s'}}$, find the sum of digits of A.
- (A) 18189 (B) 16168 (C) 9090 (D) 9099 (E) None of the above

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2. If the positive odd integers are arranged in rows of sixteen as shown below, then '2021' will appear in the mth row and nth column. Find $m \times n$.
[For example: 81 is in the 3rd row and 9th column]

1	3	5	7	9	11	13	15	17	19	21	23	25	27	29	31
33	35	37	39	41	43	45	47	49	51	53	55	57	59	61	63
65	67	69	71	73	75	77	79	81	83	85	87	89	91	93	95
								⋮							
								⋮							
								⋮							

- (A) 189 (B) 192 (C) 252 (D) 256 (E) None of the above

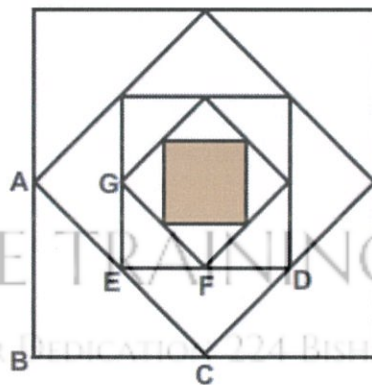
3. A number A has the same number of factors (including 1 and itself) as that of 2021 whose prime factorization is 43×47 . If the number of factors of A^2 is a perfect square, which of the following may be the number A ?
- (A) 1331 (B) 6859 (C) 1073 (D) 783 (E) None of the above

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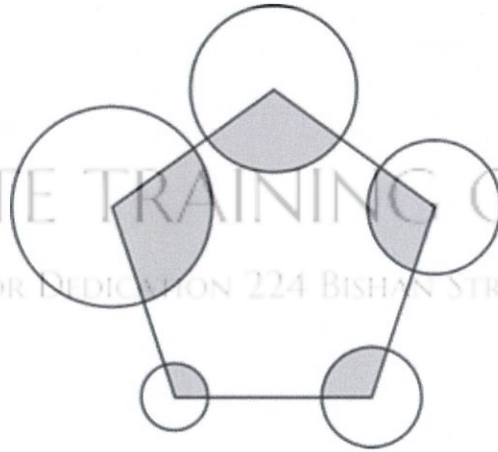
4. In the diagram, all figures are squares. Each smaller square is obtained from the bigger one by joining the mid-points. The area of the shaded square, in the diagram given below is $S \text{ cm}^2$.



How many of these statements are true?

- (i) Area of $\triangle ABC$ is $2S \text{ cm}^2$
 - (ii) Area of $\triangle CDE$ is $2S \text{ cm}^2$
 - (iii) Area of $\triangle CDF$ is $\frac{1}{2} S \text{ cm}^2$
 - (iv) Area of $\triangle EFG$ is $S \text{ cm}^2$
- (A) 1 (B) 2 (C) 3 (D) 4 (E) None of the above

5. Circles of radii 1 cm, $1\frac{1}{2}$ cm, 2 cm, $2\frac{1}{2}$ cm and 3 cm are drawn with a vertex of a regular pentagon as the centre of each circle. (A regular pentagon has all its sides equal and all its angles equal)



Taking $\pi = \frac{22}{7}$, find the total area of the shaded parts in cm^2 .

- (A) $\frac{495}{7}$ (B) $\frac{297}{14}$ (C) $\frac{550}{7}$ (D) $\frac{220}{21}$ (E) None of the above

6. Adrian and Bryan are running at constant speeds around a circular track of length 400 m. Adrian's speed is $\frac{2}{3}$ that of Bryan's. They start running from the same point and at the same time, but in opposite directions. 4 minutes later, they have met four times. Find in m/s, the difference between Bryan's speed and Adrian's speed.

- (A) $\frac{2}{3}$ m/s (B) $\frac{3}{4}$ m/s (C) 1 m/s (D) $\frac{4}{3}$ m/s (E) None of the above

7. A number is removed from a list of six consecutive integers. The sum of the remaining five numbers is 662. What is the position of the number that was removed when the six numbers are written in the increasing order?
- (A) 2nd (B) 3rd (C) 4th (D) 5th (E) None of the above

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8. The ratio of the number of marbles in three bags labelled Bag A, Bag B and Bag C is 3 : 1 : 5. 10% of the marbles in Bag C are removed and transferred to Bag B. One-third of the remaining marbles in Bag C are removed and transferred to Bag A. Now, the total number of marbles in Bag A and Bag B is 2400, find the number of marbles in Bag C at the end?
- (A) 1200 (B) 800 (C) 600 (D) 1800 (E) None of the above

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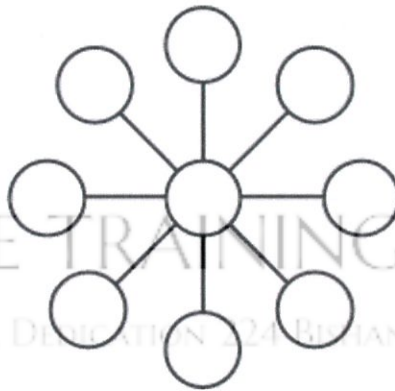
9. The digits of 972, 521 and 210 are in decreasing order whereas the digits of 279, 522 and 212 are not. How many 3-digit even numbers have their digits in decreasing order?
- (A) 60 (B) 70 (C) 84 (D) 120 (E) None of the above

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10. Nine consecutive whole numbers are written in the circles shown below, such that the sum of three numbers along each line is the same.



If 2021 is the largest of the nine whole numbers, what is the least possible sum of numbers along each line?

- (A) 6036 (B) 6051 (C) 6054 (D) 6066 (E) None of the above

11. Find the value of

$$\frac{1}{43} + \frac{1}{43 + 86} + \frac{1}{43 + 86 + 129} + \cdots + \frac{1}{43 + 86 + 129 + \cdots + 2021}$$

- (A) $\frac{47}{1032}$ (B) $\frac{49}{1032}$ (C) $\frac{1}{1032}$ (D) $\frac{47}{2064}$ (E) None of the above

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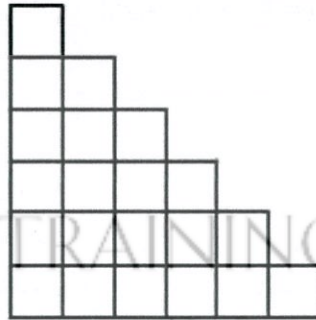
12. $S = 1^a + 2^b + 3^c + 4^d + 5^e$, where a, b, c, d, e are all positive integers and d is odd. Which of the following value for b will ensure that the last digit of S is not 1?

- (A) 24 (B) 74 (C) 99 (D) 101 (E) None of the above

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13. Alicia builds a structure with small cubes of the same size. The top view of the structure is a square. The front view and one of the side views are the same as shown below.



Alicia built this structure with the maximum number of cubes possible (she observes that adding even one cube to the structure would change the views). She wants to paint the entire structure including the base. How many faces of the cubes does she have to paint?

- (A) 120 (B) 126 (C) 156 (D) 171 (E) None of the above

14. A palindromic number is one which reads the same forwards and backwards. 272, 979 are examples of such numbers. All palindromic numbers in increasing order starting with 11 are written in a row like this: 112233445566778899101111121... In the above sequence, what is the digit in the 661st position?

- (A) 0 (B) 1 (C) 3 (D) 9 (E) None of the above

15. In Ivan's field, there are four trees forming the vertices of a square with adjacent trees 10 metres apart. Ivan asks two of his friends, Alex and Ben, for suggestions to create a triangular enclosure using the four trees as part of the fence. Here are the enclosures suggested by Alex and Ben. Both of them have used isosceles right angled triangles for their models.



- What is the ratio of the area enclosed by Alex's model : area enclosed by Ben's model?
 (A) 2:3 (B) 4:9 (C) 8:9 (D) 8:27 (E) None of the above

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16. For a whole number m , $m! = 1 \times 2 \times 3 \times \dots \times m$.

What is the remainder when $1! \times 2! + 2! \times 3! + 3! \times 4! + \dots + 2020! \times 2021!$ is divided by 18?

- (A) 2 (B) 4 (C) 12 (D) 14 (E) None of the above

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17. Find the last digit of

$$2021^3 - 2020^3 + 2019^3 - 2018^3 + \dots + 3^3 - 2^3 + 1^3$$

- (A) 4 (B) 5 (C) 6 (D) 1 (E) None of the above

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18. Player A and Player B are competing in a table tennis game. Starting from a score of 0 - 0, the two competitors in a game of table tennis earn points one at a time. As long as the competitors don't reach a score of 10 - 10, the game ends when one competitor reaches 11 points.

Samar, who is watching the game, receives a phone call when Player A has 8 points and Player B has 0 points. He steps outside to answer it. When he comes back inside, the game is over and Player A has won the game and the final score was 11 - 8. In how many different ways could Player A have reached the score 11 - 8 from the score of 8 - 0?

- (A) 9 (B) 36 (C) 48 (D) 55 (E) None of the above

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19. A large $4 \times 4 \times 4$ cube is made from identical small cubes. Some faces of the large cube have been painted. If the number of small cubes that have paint on at least one of the faces is 37, how many faces of the large cube have been painted?
(A) 2 (B) 3 (C) 4 (D) 5 (E) None of the above

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20. There are n children arranged in a circle, numbered from 1 to n in the clockwise direction and facing inwards.



They play a game of passing the parcel. The teacher calls out a "magic" number and randomly picks a child from the circle to start the game. Proceeding clockwise, the student must pass the parcel to the player who is away from them by the magic number of places. For example, if the teacher asks Student 4 to start the game and says 2 is the magic number, then the passage of the parcel will be as follows:

Student 4 \rightarrow Student 6 \rightarrow Student 8 \rightarrow ...

A round ends only when the parcel comes back to the student who started the round. Five rounds of the game were played with 4, 6, 7, 8, 9 as magic numbers respectively and the same student started the game each time. In every round, there is at least one student who does not get the parcel. What is the minimum value of n ?

- (A) 21 (B) 42 (C) 84 (D) 504 (E) None of the above

END OF PAPER

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