Mathlete Training Centre
RIPMWC 2019

1. A, B, C and D are 4 numbers such that $A-B=2017$, $C-B=2018$ and $C-D=2019$, find the value of

$$\frac{A-D}{(A-C)(B-D)}$$

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2. How many numbers which are greater than 200 can be formed by using digits from 2019 at most once?

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3. A bag contains only yellow, green and red marbles. The ratio of the number of yellow marbles to the number of green marbles is 4:7. The ratio of the number of green marbles to the number of red marbles is 2:3. There are 40 yellow marbles in the bag. Find the total number of green and red marbles in the bag.

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4. Given that $(2\frac{3}{5}$ - $\frac{7}{3}$ (8.5 - x)) $\div$ (1 $\div$ (3.05 + $4\frac{9}{20}$)) = 2, find the value of x.

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5. Calculate

$$\frac{2018^2}{673} - \frac{2017^2}{673} + \frac{2016^2}{673} - \frac{2015^2}{673} + ... + \frac{2^2}{673} - \frac{1^2}{673}$$

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6. How many of the first 729 positive integers are perfect squares, cubes or both?

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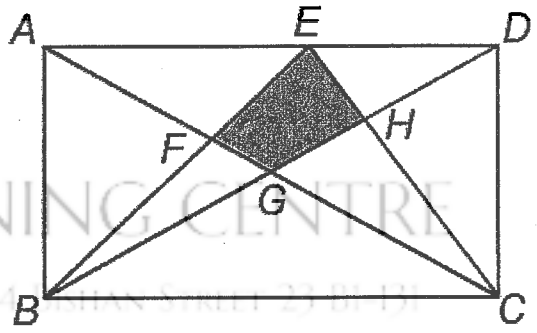
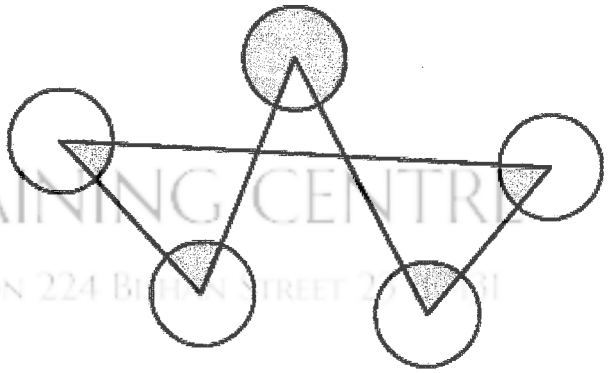
7. In the diagram below, ABCD is a rectangle with BC = 7 cm and CD = 3 cm. If the sum of area of triangle ABF and area of triangle CDH is $7cm^2$, find the area of the quadrilateral EFGH in cm^2.



8. The diagram shows 5 circles, each with radius 7 cm. By taking $\pi = \frac{22}{7}$, find the total area of the shaded parts.



9. Two trains are travelling in the same direction along parallel tracks. Train A is 300 metres long and travelling at 75 km/h. Train B is 400 metres in length and 8 km ahead of Train A (this distance is measured from the back of Train B to the front of Train A). Twenty minutes later, Train A is 8 km ahead of Train B. Assuming that the speed of each train is constant and distance between 2 tracks is very small, find the speed of Train B in km/h.

10. How many different triangles can be formed by connecting 3 of the 11 points below, where 5 points lie on a line while the other 6 points lie on a different line.

11. If $A = 9 + 999 + 99999 + ... + \underbrace{999...999}_{(2017\ "9")} + \underbrace{999...999}_{(2019\ "9")}$, find the sum of the digits of A.

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12. Calculate

$$(\frac{1}{31}+\frac{1}{42}+\frac{1}{49}+\frac{1}{51})\times(\frac{1}{42}+\frac{1}{49}+\frac{1}{51}+\frac{1}{65})-(\frac{1}{31}+\frac{1}{42}+\frac{1}{49}+\frac{1}{51}+\frac{1}{65})\times(\frac{1}{42}+\frac{1}{49}+\frac{1}{51})$$

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13. Numbers 1, 2, 3, ..., 8, 9, 10 are placed in a bag and drawn at random with replacement. How many ways can three numbers be drawn whose sum is 13?

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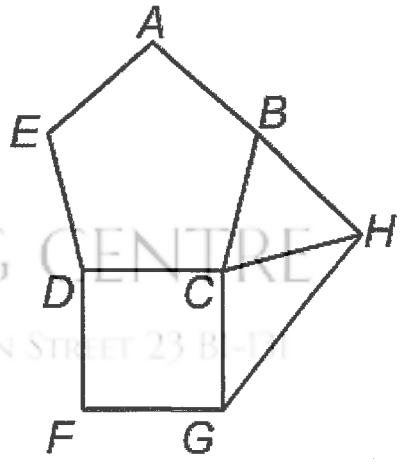
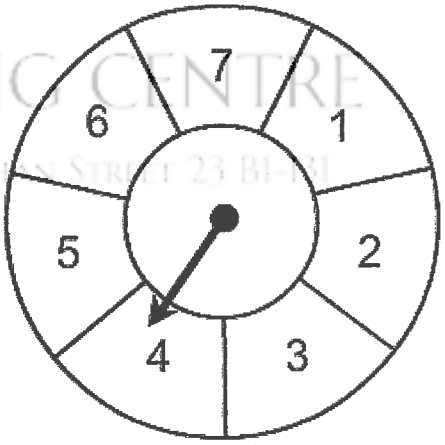
14. In the above diagram, ABCDE is a regular pentagon (all its sides are equal and all its angles are equal). CDFG is a square and $\triangle BCH$ is a equilateral. Find angle CGH.

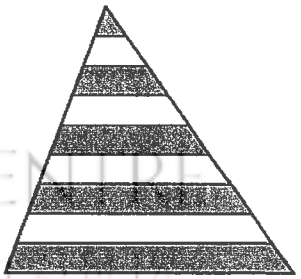


15. Dave is playing a game called Mod Seven. At the beginning of the game, the arrow points to one of the 7 numbers. On each turn, the arrow is rotated clockwise by the number of spaces indicated by the arrow at the beginning of the turn. For example, if Mod Seven starts with arrow pointing at 4, then on the first turn, the arrow is rotated clockwise 4 spaces so that it now points at 1. The arrow will then move 1 space on the next turn, and so on. If the arrow points at 6 after the 2019 th turn, at which number did the arrow point after the first turn?



16. Mr Chan has a certain number of pencils to give to pupils in the 3 classes he teaches X, Y and Z.
If all the pencils are distributed to classes X, Y and Z, each pupil will get 42 pencils.
If all the pencils are distributed to class X only, each pupil will get 105 pencils.
If all the pencils are distributed to class Y only, each pupil will get 98 pencils.
If all the pencils are distributed to class Z only, how many pencils will each get?

17. The triangle above is divided into nine stripes of equal width and each strip is parallel to the base of the triangle. If the total area of the unshaded strips is $88cm^2$, find the total area of the shaded strips in cm^2



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18. A is the largest integer such that 14A has exactly 2019 digits. Counting from right to left, what is the 100^{th} digit of A?

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19. John writes a number with 729 digits on the blackboard, each digit is either 1 or 2. Esther creates a new number with 2187 ($= 3 \times 729$) digits by replacing each 1 with 112 and each 2 with 111. For example, if John's number begins with 2112, then Esther's number would begin with 111112112111. After Esther finishes writing her number, she notices her leftmost 729 digits in her number and John's number are the same. How many times do five 1s occur consecutively in John's number?

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20. Find the value

$$\frac{1}{7}+\frac{1}{7^2}+\frac{2}{7^3}+\frac{3}{7^4}+\frac{5}{7^5}+\frac{8}{7^6}+\frac{13}{7^7}+\ldots$$

where the numerator of each fraction comes from the Fibonacci sequence 1, 1, 2, 3, 5, 8, 13, 21, ... (the first 2 terms are both 1 and from the 3^{rd} term onwards, it is the sum of the 2 terms just before it)

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