Mathlete Training Centre RIPMWC 2017

1. Find the sum $\frac{1}{2} + (\frac{1}{3} + \frac{2}{3}) + (\frac{1}{4} + \frac{2}{4} + \frac{3}{4}) + (\frac{1}{5} + \frac{2}{5} + \frac{3}{5} + \frac{4}{5}) + (\frac{1}{2017} + \frac{2}{2017} + \dots + \frac{2015}{2017} + \frac{2016}{2017}).$

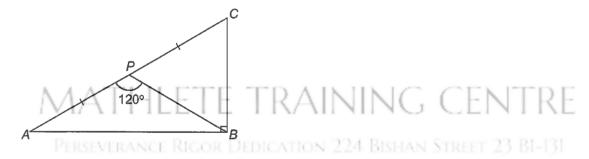
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2. Triangle ABC is a right-angled triangle. P is the midpoint of AC and $\angle APB = 120^{\circ}$. Given that AC = 5, what is the length of BC?



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3. Calculate $\frac{2017\frac{7}{8} \times 3\frac{2}{3} - 2016.875 \times \frac{11}{3}}{(2017\frac{1}{3} - 2016.875) \times 17} \times 3\frac{23}{32}$.

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- 4. The diagram below has 10 boxes. The numbers in the second and ninth boxes are 177 and 130 respectively. Find the first number in the box if:
 - i) The sum of all numbers in the boxes is 2017 and
 - ii) The sum of the numbers in any three consecutive boxes is the same.

177		130

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5. How many 3-digit and 4-digit numbers can be formed using each of the 4 digits from 2017 at most once?

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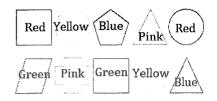
6. In one morning, a ferry travelled from Island A to Island B and another ferry travelled from Island B to Island A at a different speed. They started at the same time and met first time at 8.30 am. The two ferries then sailed to their destinations, stopped for 15 minutes and returned. The two ferries met again at 9.23 am. Suppose the two ferries travelled at a uniform speed throughout the whole journey, what time did the two ferries start their journey?

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7. Grace and Michelle are playing a game with their teacher. The teacher places ten counters of different colours and shapes on the table in front of them as shown in the diagram below:



The teacher tells them both that she has hidden a coin under one of the counters. She then tells Grace the shape of the counter she has hidden it under, and tells Michelle the colour of the counter. She proceeds to ask them in turn whether they know which counter the coin is hidden under. They can only reply yes or no, but they can listen to each other's answers. Their answers are as follows:

Grace: "No". Michelle: "No". Grace: "No".

Michelle: "Yes".

Assuming they answer "Yes" only when they are 100% sure and "No" otherwise, which counter is the coin hidden under?

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8. The figure below is made up of four overlapping circles, each with a radius of 1 cm, which are centred at the points A, B, C and D respectively. What is the perimeter of the shaded region in cm? [Take $\pi = \frac{22}{T}$]

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9. Calculate $\frac{5}{36} + \frac{7}{144} + \frac{9}{400} + \frac{11}{900} + \frac{13}{1764} + \frac{15}{3136}$.

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- 10. Let $x = 2017^{2017} + 2017$. How many of the following statements are true?
 - (I) The remainder when you divide x by 5 is 2.
 - (II) The remainder when you divide x by 6 is 2.
 - (III) The remainder when you divide x by 7 is 2.
 - (IV) The remainder when you divide x and 8 is 2.

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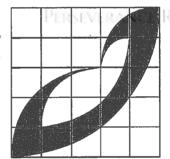
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11. John chooses three different numbers from 2017, 2018, 2019, ..., 2033, 2034, 2035. How many wavs can this be done if the sum of these 3 numbers is to be divisible by 4?

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12. The diagram below is made from quarter circles on a square grid with divisions 1 cm apart. What is the total area of the shaded region in cm²? [Take $\pi = \frac{22}{7}$]



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13. If $S = 2 + 22 + 222 + 2222 + \cdots + 222 \cdot \cdot \cdot \cdot 222$, what is the sum of the last 5 digits of S?

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14. ABCD is a square with side length of 2 units, and P is the midpoint on CD. What is the total area of the shaded region, which is made up of a square and 2 triangles, in units²?



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15. A pirate has buried his treasure chest under a beach, and he drew a map of the beach in the form of a grid where each section is numbered from 1 to 49 as shown in the diagram below:



The shape of his treasure chest is made up of squares like those shown in the choices below which are aligned perfectly with the grid above when buried. To find his way back to the treasure, he made a note of the sum of all the grid numbers that sit above the chest once buried. If the sum is 86, which of the following is a possible shape of the chest?









None of the above

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16. Fractions in the form $\frac{a}{b}$ are created such that a and b are positive integers and a+b=425. How many such fractions are there which are less than 1 and which cannot be simplified further (that is to say, a and b have no common factors others than 1)?

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17. Let $A = 1 \times 2 \times 3 \times \cdots \times 2008 \times 2009 \times 2010$. Reading the digits of A from right to left, what is the first non-zero digit?

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18. Pencils of the same shape and size are placed in a big container. There are 9 different colours and 75 pencils in each colour. What is the minimum number of pencils that must be drawn from the container to ensure that the pencils are of at least 5 different colours with at least 10 pencils for each colour?

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19. Consider the three numbers 2017, 2758 and 3670. When they are divided by a number A, the remainder is the same for each of the three numbers. What is the sum of the possible values of A?

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20. Box A contains four times as many coins as Box B. 6% of the coins in Box A are removed and transferred to Box C. 20% of the coins in Box B are removed and transferred to Box C. There are now 1220 coins in Box C, which is 22% more than the original number of coins in Box C. How many coins were there in box A at the beginning?

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