Mathlete Training Centre
RIPMWC 2016

1. What is $\dfrac{1.02+2.04+4.08+8.16+16.32+32.64}{3.06+6.12+12.24+24.48}$ in its simplest form?

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2. Minnie, Nelson, Omar and Petra are arranging a Secret Santa gift exchange: they will each buy one gift for another member and the group, and each receive one gift. They are not allowed to receive their own gift. How many ways can they exchange gifts?

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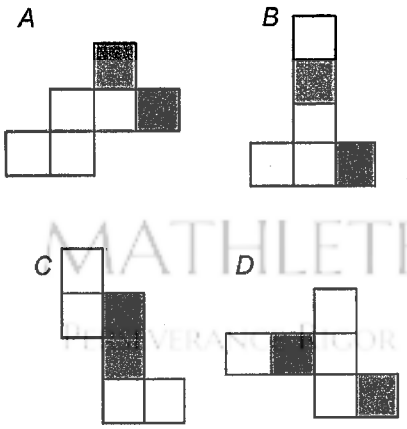
3. When these nets are folded to make cubes, which (if any) will have the two coloured faces directly opposite each other?



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4. How many perfect squares divide 2016?

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5. Christopher has a list of all the numbers from 1 to 100 (including both 1 and 100). He starts by deleting one of the numbers from the list, and proceeds as follows: If any multiples of the last number deleted are still on the list, delete the largest such multiple. Otherwise delete the largest remaining factor of the last number deleted. Keep repeating this step until no more numbers can be deleted.
   If the first number Christopher deletes is 71, which is the last number he will delete?

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6. Caroline, Dylan and Eddie are playing a game. At the beginning of the game, they each have a number of counters. The counters are shared out between the players in the ratio 5:4:3. By the end of the game, the total number of counters is the same but the number of counters each player has is in the ratio 4:3:2. If one of them won 8 counters over the course of the game, how many counters did the three players have in total at the beginning of the game?

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7. For a whole number n, $n! = n \times (n-1) \times (n-2) \times \cdots \times 3 \times 2 \times 1$. What is the remainder when $1! + 2! + 3! + \cdots + 2015! + 2016!$ is divided by 15?

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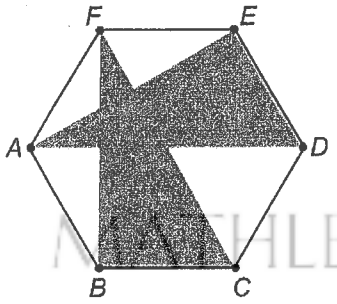
8. What proportion of the regular hexagon $ABCDEF$ is shaded?



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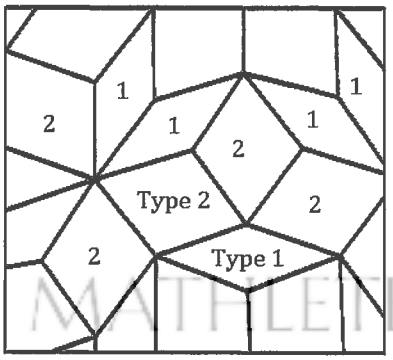
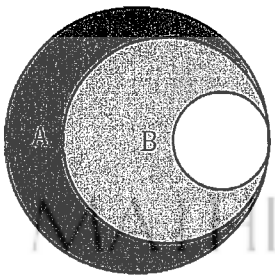
9. Colin forms a $32 \times 9 \times 7$ cuboid from 2016 unit cubes, and paints the exterior of this cuboid red. How many of the 2016 unit cubes have 0 or 2 of their faces painted red?

10. The image below shows part of a floor which is tiled by two different kinds of rhombus, one narrow Type I rhombus, the other wide Type 2 rhombus. What is the difference between the biggest angle of the Type 2 rhombuses and the smallest angle of the Type 1 rhombuses?



11. The figure below is formed of three circles with integer radius which intersect at a common point. The largest circle has a radius of 17 units. Given that area A is the same as area B, what is the sum of the radii of all three circles?



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12. A square with side length 1 cm is removed from each corner of a square with side length 6 cm. What is the area of the largest square that can be drawn in the remaining shape?

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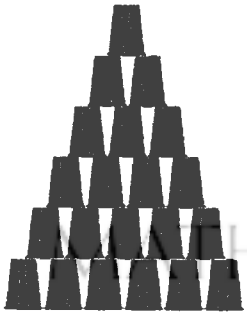
13. In some cultures it is considered unlucky when the 13^{th} day of a month falls on a Friday. What is the most number of times this could happen in one calendar year (which could be a leap year)?

14. Alice and Bob are the world's foremost cup stackers - people who build triangular towers out of cups. Alice had exactly enough cups to build a tower 50 levels tall, and Bob has exactly enough cups to build a tower 38 levels tall. how many levels tall is the tallest tower they could build using all their cups together?



15. Let $x = 7^a + 21^b$, where a and b are some positive integers. Which of the following could not be the last two digits of x?

20 22 24 28 None of the above

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16. For how many 4-digit numbers is the sum of the two middle digits equal to the sum of the two outer digits?

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17. Joanna and Kai's car has broken down, so their daughter Layla comes to fetch them in her own car. Only two people (including the driver) can fit into Layla's car, so they use the following procedure: Layla drives Joanna some distance in the car, while Kai follows them on foot. Then Layla drops Joanna, who will continue on foot and turns back, driving until she meets Kai. She picks up Kai and they drive onwards until they catch up with Joanna. Then Kai and Joanna swap, and the process continues for 3 more times until they all reach home. If Layla drives at a constant speed of 50 km/h and Joanna and Kai both walk at a constant speed of 4 km/h, for what fraction of the journey will Layla be alone in her car?

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18. How many 5-digit numbers are there whose product of digits is 2016?

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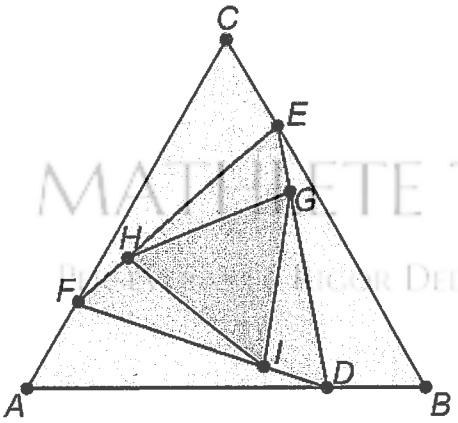
19. ABC, DEF and GHI are equilateral triangles. D lies $\frac{3}{4}$ of the way along AB, E lies $\frac{3}{4}$ of the way along BC, and F lies $\frac{3}{4}$ of the way along CA. Similarly, G lies $\frac{3}{4}$ of the way along DE, H lies $\frac{3}{4}$ of the way along EF and I lies $\frac{3}{4}$ of the way along FD. What is the ratio of the area of triangle GHI to the area of triangle ABC?



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20. Toby and Sam are playing a game. They have two bags of sweets. Bag A contains three sweets, and bag B contains six sweets. They take turns to remove at least one sweet according to this rule:
On his turn, a player may take either as many sweets as he likes from one bag (including all the sweets in the bag), or the same number of sweets from both bags. The person who takes the last sweet is the winner. If Toby goes first, which of these 1st moves guarantees that he will win?
i) Remove all sweets from bag A
ii) Remove one sweet from bag A
iii) Remove one sweet from bag B
iv) Remove two sweets from bag B
v) None of the above.