Mathlete Training Centre
Round 1 RIPMWC open

2015 RIPMWC open round 1

1. If $A \otimes B = \frac{B}{A} + \frac{2A}{B} + 2015$, find $(5 \otimes 10) - (10 \otimes 15)$

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2. Find the sum $4 + 8 + 16 + 32 + 64 + 128 + 256 + 512 + 1024 + 2048$

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3. Which of the following numbers has the same number of divisors (including 1 and the number itself) as 2015? (Note that $2015 = 5 \times 13 \times 31$)

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4. What is the value of $\dfrac{2002.2002 + 20022002 + 200220022002}{2015.2015 + 20152015 + 201520152015}$?

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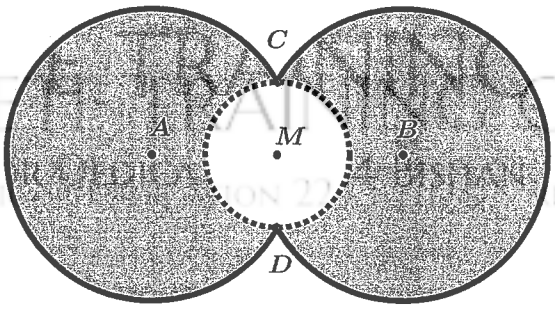
5. Two circle of equal radius with centres at A and B intersect at the points C and D. A further circle is drawn passing through both C and D, with its centre which is the midpoint of A and B and which is also mid-point of C and D. Given that the smaller circle has half the radius of one of the bigger circle, what is the ratio of the outer perimeter (solid line) of the shaded area to the inner perimeter (dotted line)?



6. After a football game, each of the 11 players in a football team puts both their boots in one bag. The 4 defenders in the team wear red boots but the other player wear black boots. You begin taking boots out of the bag at random, one at a time, without replacement. What is the largest number of boots you might have to take out to get at least one pair of red boots worn by a defender?

7. 50 boxes are arranged in a circle, and numbered 1 to 50. A boy walks around the circle, placing 1 cent in box number 1, 2 cents in box number 2, 3 cents in box number 3, and so on till he gets back to where he started. He continues walking, and on the second time round, he stops at every even numbered box, and removes the money in it. On the third time round, he stops at every box which is a multiple of 3. If the box contains money, he removes it, but if it is empty he replaces the original amount. On the fourth time round, he does the same with every box which is a multiple of 4. If he continues in this way, what will be the total amount left in the boxes after he has walked around the circle 50 times?

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8. As shown below, Aaron writes down all the multiples of 5 from 5 to 2015 inclusive to form a number

$$510152025\ldots\ldots20102015$$

How many digits are there in this number?

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9. In the time it takes Ben to walk 11 steps, Charlie walks 9 steps. If Charlie covers the same distance in 5 steps as Ben does in 7 steps, what is the ratio of the speed of Ben to the speed of Charlie?

10. I pick a whole number. If it is even, I divide it by 2, if it is odd then I multiply it by 3 and then add 5. I then repeat this process with the new number formed.
Starting with the number 15, I form the sequence

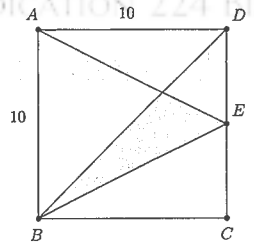
$$15 \to 50 \to 25 \to 80 \to 40 \to 20 \ldots$$

Consider the sequences starting with the numbers 1 to 10. What is the smallest number which does not appear in any of the above sequences?

11. A staircase consists of 8 steps. John walks from the bottom to the top, each time climbing 1, 2, or 3 steps. How many different ways can he climb the staircase?

12. In the figure below, $ABCD$ is a square of length 10 cm and E is the mid-point of CD.

Find $\dfrac{\text{area of the shaded region}}{\text{area of the square } ABCD}$



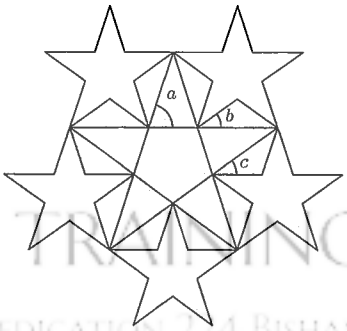
13. Find the remainder when the number 201020112012000020132014 2015 is divided by 88.

14. Two different integers are selected from 1 to 19 inclusive. In how many of these combinations of 2 numbers are their sum a multiple of 3?

15. Peter and Ryan set off for school together and must arrive at their school 25 km away at the same time. They share the use of the only available bicycle. Ryan sets out riding at 10 km/h, leaves the bicycle and walks at 6 km/h. Peter walks at 5 km/h, reaches the bicycle and rides at 12 km/h. Find the distance in km for which Ryan rides the bicycle.

16. Find the value of $\frac{1}{31}+\frac{1}{31+62}+\frac{1}{31+62+93}+\cdots+\frac{1}{31+62+93+\ldots+2015}$

17. A pattern is formed using 6 regular 5-pointed stars as shown. What is the relation between a, b and c?



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18. There are 4 bags labelled A, B, C and D. 8 identical coins are then distributed into the 4 bags such that each bag must contain at least one coin. Find the number of possible ways that this can be done.

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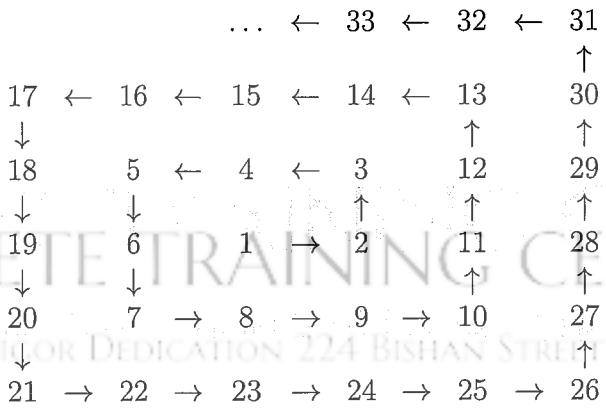
19. If the whole numbers are written in an anticlockwise spiral on a square grid as shown, which number appears directly above 2015?



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20. Jack and Ken play a game of picking up coins from a pile of 2015 coins. They take turn to pick up coins with Jack starting first. The rules of the game require each person to pick up only $1, 2, 3, 4, 5, 6, 7, 8, 9, 10$ or 11 coins at each turn and the person to pick up the last coin is the loser. How many coins should Jack pick up at his first turn to ensure that he will be the winner?

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