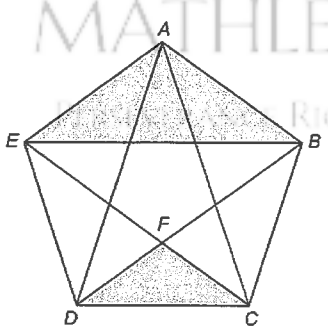
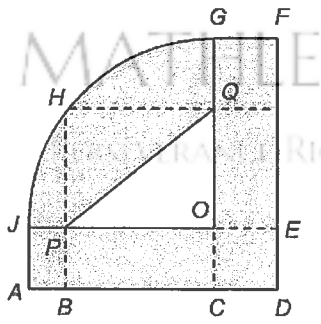
Mathlete Training Centre
2014 Open Round 1

RIPMWC

1) A star is made by connecting the vertices of a rectangular pentagon ABCDE. The area of the pentagon ABCDE is $6.882cm^2$, and the area of triangle ABE is $1.902cm^2$. What is the area of triangle CDF in cm^2



2) The sum of the perimeters of shapes OPQ and ABCDEFGHIJ is 9.95m. OGJ is quarter-circle of radius 1.4m and GF = AJ = 0.4m. Taking $\pi = \frac{22}{7}$, find the perimeter of rectangle OQHP in m.



3) What is the measure of the angle formed between the hour hand and the minute hand of a clock at 9:24?

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4) How many solutions are there to $\frac{1}{a} + \frac{1}{b} = \frac{8}{15}$ with a and b being whole numbers and $a < b$?

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5) Calculate $\cfrac{2014}{2013-\cfrac{2012}{2011-\cfrac{2010}{\ldots 5-\cfrac{4}{3-\frac{2}{1}}}}}$

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6) How many ways are there to make \$80 using some combination of \$5, \$10*and*\$20 notes?

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7) What is the last digit of
$2014^{2014} - 2014^{2013} + 2014^{2012} - 2014^{2011} + ... - 2014^3 + 2014^2 - 2014^1$?

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8) After John walked $x\%$ of the distance from his home to his school at a constant speed, he turned around and walked home, got his bicycle and cycled to his school and back home. John cycles three and a half times faster than he walks. Find the largest possible value of x so that returning home to get his bicycle did not take more time than him walking all the way to and from his school without his bicycle.

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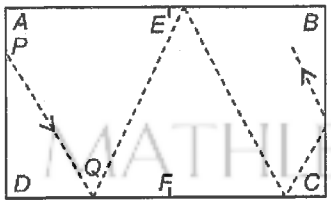
9) The number 95__94775998 is divisible by 198. What is the missing digit?

10) A pair of positive integers (a, b) is said to be an 'awesome pair' if $a^2 - b^2$ is positive and is a factor of 2014. How many awesome pairs are there with both a and b less than 100?

11) A snooker table ABCD is 370cm by 180cm in size. There are pockets in the 4 corners and in the middle of the longer edges (ie. at A, B, C, D, E, F). When a ball is hit, you can assume that it bounces off the table at the same angle as it hits.
A ball is hit towards edge DC from point P and goes into pocket F after 5 bounces. Given that P is 50cm from A, what is the distance DQ to the nearest cm?



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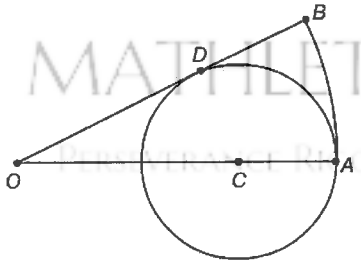
12) In the diagram above, OAB is a circular sector with $OA = OB$ and $\angle AOB = 30°$. A circle passing through A is drawn with centre C on OA, touching OB at a point D. If the area of the circular sector OAB is $7cm^2$, find the area of the circle with its centre at C and radius CA in cm^2.



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13) Calculate the sum
$1+2+3-4+5+6+7-8+9+10+11-12+...+2009+2010+2011-2012+2013+2014$

14) If $a+b=\dfrac{1}{\frac{1}{a}+\frac{1}{b}}$ then what is $(1\times 2)+(2\times 3)+(3\times 4)+...+(2013\times 2014)$?

15) Martha writes down a list of numbers where each number is the sum of the previous two numbers on the list. The first numbers she writes down are both 1s, and so the third number she writes is $1 + 1 = 2$. If she divides the 2014th number on the list by 7, what is the remainder?

16) Esther has 25 coins in a single pile and she is trying to split them up so that each coin ends up in a pile by itself. Every time she splits a pile into 2 sub-piles, one with a coins and the other with b coins, she will get $(a \times b)$ points added to her 'score'. (E.g. from a pile with 5 coins, which she splits into a sub-pile of 2 coins and another pile of 3 coins, she gets 6 points added to her score). From a starting score of 0 points, what is the largest score she can attain?

17) 7 women are standing in a row. Each woman has 3 hats, one red, one blue and one yellow. The woman in the middle (4th in the row) has a black hat and a white hat in addition to the 3 hats each has. How many combinations of hats can they wear if no 2 women next to each other wears a hat of the same colour?

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18) x is the smallest whole number whose digits add up to 2014. What is the sum of the first and last digits of x^2?

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19) How many perfect squares less than 1000 can be written as a sum of 2 consecutive numbers, and also as a sum of 3 consecutive numbers?

20) How many whole numbers n not more than 2014 are there such that $\frac{n}{2014}$ is a fraction in its simplest form (ie. n and 2014 have no common factor > 1)?