

Mathlete Training Centre  
Round 1 RIPMWC open

2012 RIPMWC open round 1

---

1.

If all the positive integers with leftmost digit 8 are listed in ascending order:

8, 80, 81, 82, 83, 84, ... . What is the 2012<sup>th</sup> integer in the list?

MATHLETE TRAINING CENTRE

PERSEVERANCE RIGOR DEDICATION 224 BISHAN STREET 23 BI-131

MATHLETE TRAINING CENTRE

PERSEVERANCE RIGOR DEDICATION 224 BISHAN STREET 23 BI-131

2.

Calculate  $\frac{2 \times 6 \times 10 + 3 \times 9 \times 15 + 4 \times 12 \times 20 + 5 \times 15 \times 25 + 6 \times 18 \times 30}{2 \times 4 \times 6 + 3 \times 6 \times 9 + 4 \times 8 \times 12 + 5 \times 10 \times 15 + 6 \times 12 \times 18}$

MATHLETE TRAINING CENTRE

PERSEVERANCE RIGOR DEDICATION 224 BISHAN STREET 23 BI-131

3.

At B&W Restaurant, the cost for a lunch consisting of 3 hamburgers, 7 cups of coffee and one apple pie adds up to \$12.60 and the cost of a lunch consisting of 4 hamburgers, 10 cups of coffee and one apple pie adds up to \$16.85. What would be the cost of one hamburger, one cup of coffee and one apple pie at this restaurant?

MATHLETE TRAINING CENTRE

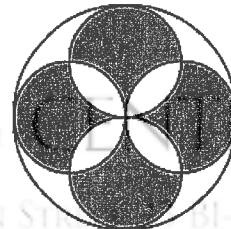
PERSEVERANCE RIGOR DEDICATION 224 BISHAN STREET 23 BI-131

MATHLETE TRAINING CENTRE

PERSEVERANCE RIGOR DEDICATION 224 BISHAN STREET 23 BI-131

4.

As shown in the figure, 4 identical smaller circles are constructed symmetrically in a bigger circle, with each smaller circle touching the bigger circle. If the radius of the bigger circle is 7 cm and the radius of each smaller circle is 3.5 cm, then the shaded region, in  $\text{cm}^2$ , is



MATHLETE TRAINING CENTRE

PERSEVERANCE RIGOR DEDICATION 224 BISHAN STREET 23 BI-131

5. The least common multiple of 3 numbers  $x$ , 12 and 2012 is 6036. Find the number of positive integers  $x$  with this property.

MATHLETE TRAINING CENTRE

PERSEVERANCE RIGOR DEDICATION 224 BISHAN STREET 23 BI-131

MATHLETE TRAINING CENTRE

6. Two swimmers, at opposite ends of a 30-metre pool, start to swim the length of the pool, one at 1 metre per second, the other at  $\frac{2}{3}$  metre per second. They swim back and forth for 12 minutes. Assuming no loss of time at the turns, find the number of times they pass each other.

MATHLETE TRAINING CENTRE

PERSEVERANCE RIGOR DEDICATION 224 BISHAN STREET 23 BI-131

7.

John calculates  $\frac{2}{1} + \frac{3}{2} + \frac{4}{3} + \dots + \frac{101}{100} + \frac{102}{101}$  and

Esther calculates  $\frac{1+2}{3} + \frac{4+5}{6} + \frac{7+8}{9} + \dots + \frac{298+299}{300}$ . The sum of their answers is between

MATHLETE TRAINING CENTRE

PERSEVERANCE RIGOR DEDICATION 224 BISHAN STREET 23 BI-131

MATHLETE TRAINING CENTRE

PERSEVERANCE RIGOR DEDICATION 224 BISHAN STREET 23 BI-131

8.

A game is played with identical coins according to the following rule.

In each by round, the player with the most coins gives one coin each to the other 2 players and also place one coin into the discard pile. The game ends when one player runs out of coins. Players X, Y and Z start with 20, 19 and 18 coins, respectively. How many rounds will the game last?

MATHLETE TRAINING CENTRE

PERSEVERANCE RIGOR DEDICATION 224 BISHAN STREET 23 BI-131

9.

A contractor estimated that worker A would take 10 hours to build a given wall and worker B would take 11 hours for the same job. However, he knew from experience that when they work together, their combined output fell by 48 bricks per hour. In a hurry to finish off the job, he put both men to work together and they took exactly 6 hours to build the wall. Find the number of bricks in the wall.

MATHLETE TRAINING CENTRE

PERSEVERANCE RIGOR DEDICATION 224 BISHAN STREET 23 BI-131

MATHLETE TRAINING CENTRE

PERSEVERANCE RIGOR DEDICATION 224 BISHAN STREET 23 BI-131

10.

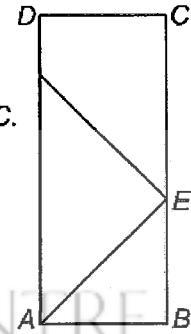
By using 1, 2, 3 or 4 digits from the number 2012, how many positive numbers less than 2210 can be formed?

MATHLETE TRAINING CENTRE

PERSEVERANCE RIGOR DEDICATION 224 BISHAN STREET 23 BI-131

11.

On a rectangular table  $ABCD$  of length 7 units and breadth 3 units, a ball is rolled from point  $A$  at an angle of  $45^\circ$  towards point  $E$ , and bounces off  $CD$  at an angle of  $45^\circ$ . The ball continues to bounce off the sides at  $45^\circ$  until it reaches  $C$ . How many times has the ball bounced?



MATHLETE TRAINING CENTRE

PERSEVERANCE RIGOR DEDICATION 224 BISHAN STREET 23 BI-131

MATHLETE TRAINING CENTRE

PERSEVERANCE RIGOR DEDICATION 224 BISHAN STREET 23 BI-131

12.

For any two numbers  $x$  and  $y$ ,  $x * y$  means  $x + y - \frac{2012}{335}$ .

Calculate  $\frac{2 * 4 * 6 * \dots * 2010 * 2012}{2012}$ .

MATHLETE TRAINING CENTRE

PERSEVERANCE RIGOR DEDICATION 224 BISHAN STREET 23 BI-131

13.

The number  $\underbrace{55\dots5}_{20 \text{ digits}} \underbrace{X99\dots9}_{20 \text{ digits}}$  consists of 41 digits, the first 20 being 5 and the last 20 being 9.

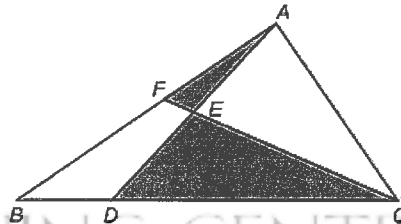
If the number is divisible by 7, the missing digit  $X$  is

MATHLETE TRAINING CENTRE

PERSEVERANCE RIGOR DEDICATION 224 BISHAN STREET 23 BI-131

14.

In  $\triangle ABC$ ,  $D$  is a point on  $BC$  such that  $DC = 3BD$ .  $F$  is a point on  $AB$  such that the intersection point on  $AD$  and  $CF$ ,  $E$ , satisfies  $AE = ED$ . If the area of  $\triangle ABC = 1 \text{ unit}^2$ , find the area of the shaded region.



MATHLETE TRAINING CENTRE

PERSEVERANCE RIGOR DEDICATION 224 BISHAN STREET 23 BI-131

15. At a party, there were  $c$  children,  $w$  women and  $m$  men, where  $c$ ,  $w$  and  $m$  are at least 2,  $c$  is less than  $w$  and  $w$  is less than  $m$ . Every person shook hand with every other person. The sum of the number of handshakes between children, the number of handshakes between women and the number of handshakes between men was 57. How many handshakes were there altogether?

MATHLETE TRAINING CENTRE

PERSEVERANCE RIGOR DEDICATION 224 BISHAN STREET 23 BI-131

MATHLETE TRAINING CENTRE

PERSEVERANCE RIGOR DEDICATION 224 BISHAN STREET 23 BI-131

16. Find the value of the expression
- $$2! \times 4 - 3! \times 5 + 4! \times 6 - 5! \times 7 + \dots + 2010! \times 2012 - 2011! \times 2013 + 2012!$$
- where  $n! = 1 \times 2 \times 3 \times \dots \times n$

MATHLETE TRAINING CENTRE

PERSEVERANCE RIGOR DEDICATION 224 BISHAN STREET 23 BI-131



17.

The time now is between 10 00 and 11 00. Five minutes from now, the minute hand of a clock will be exactly opposite the place where the hour hand was three minutes ago.

Find the exact time now.

MATHLETE TRAINING CENTRE

PERSEVERANCE RIGOR DEDICATION 224 BISHAN STREET 23 BI-131

MATHLETE TRAINING CENTRE

PERSEVERANCE RIGOR DEDICATION 224 BISHAN STREET 23 BI-131

18.

Ninety nine lines parallel to the base of a triangle divide the other 2 sides each into 100 equal parts and the area into 100 distinct parts. If the area of the largest of these parts is  $5.97 \text{ cm}^2$ , find the area of the original triangle in  $\text{cm}^2$ .

MATHLETE TRAINING CENTRE

PERSEVERANCE RIGOR DEDICATION 224 BISHAN STREET 23 BI-131

19.

When 1 litre of water is added to a mixture of alcohol and water, the new mixture is  $33\frac{1}{3}\%$  alcohol. When 1 litre of alcohol is added to the new mixture, the result is  $42\frac{6}{7}\%$  alcohol.

Find the percentage of alcohol in the original mixture.

MATHLETE TRAINING CENTRE

PERSEVERANCE RIGOR DEDICATION 224 BISHAN STREET 23 BI-131

MATHLETE TRAINING CENTRE

PERSEVERANCE RIGOR DEDICATION 224 BISHAN STREET 23 BI-131

20.

A bag contains some blue, red and white balls. The number of blue balls is at least half the number of white balls. The number of blue balls is at most one third the number of red balls. The total number of white and blue balls is at least 58. Find the minimum number of red balls.

MATHLETE TRAINING CENTRE

PERSEVERANCE RIGOR DEDICATION 224 BISHAN STREET 23 BI-131