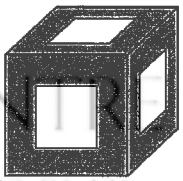
Mathlete Training Centre
Round 1 RIPMWC open

2009 RIPMWC open round 1

1.

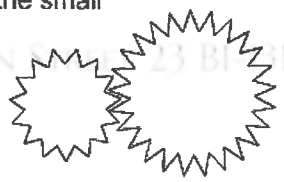
The diagram shows a cube of side 2 cm. A hole of uniform cross-section of 1 cm × 1 cm square is drilled through at the centre of each surface of the cube. Each edge of the hole is parallel to the corresponding edge of the cube. What is the surface area of the resulting solid?



2.

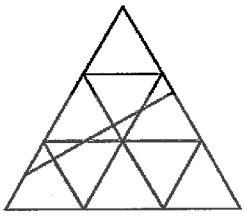
The larger gear has 25 teeth, and the smaller has 15 teeth. If the larger gear rotates through an angle of 90°, through what angle measure does the small gear rotate?

(The picture illustrates how the wheels move together but it is not accurate in terms of the number of teeth)



3.

How many triangles are there in the following figure?



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4.

Find the value of $\frac{191919}{979797} \div \frac{910910}{970970} \div \frac{19001900}{97009700}$

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5. A housekeeper has 20 keys to open 20 rooms in a hostel. He knows that each key can open only one room, but he does not know which key opens which room. If he wants to open all the 20 rooms, what is his maximum number of tries?

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6. The cost of a magazine is increased by 10% this year. With the selling price remaining the same, the profit for each magazine is decreased by 40% this year. But the number of magazines sold is increased by 80%. What is the percentage increase in the total profit from the sale of the magazine this year?

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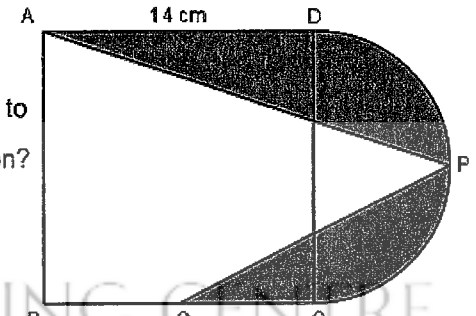
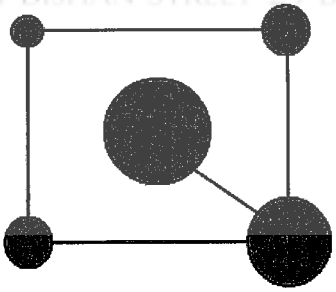
7.

The diagram shows a square ABCD of side 14 cm. Q is the mid-point of BC. CPD is the semicircle with CD as the diameter. Given that length of arc DP is equal to the length of arc PC, what is the area of the shaded region? (Take $\pi = \frac{22}{7}$)



8.

The diagram shows five discs, all of different sizes, connected by five line segments. Three colours, green, black and white is available to colour these discs. In how many different ways is it possible to colour all five discs if the discs that are connected by a line segment are to have different colours?



9.

The sum of ages of a group of people is 3075. The youngest is at least 30 years old and the oldest is not more than 80 years old. If there are at most 3 people of the same age for each age, find the minimum number of people more than 50 years old.

10.

Seven light bulbs *A*, *B*, *C*, *D*, *E*, *F* and *G* are each equipped with a pull-string switch. A pull-string switch works as follows:

- if the light bulb is switched on and you pull the string, the light bulb will be switched off.
- if the light bulb is switched off and you pull the string, the light bulb will be switched on.

Initially, light bulbs *A*, *C*, *E* and *G* were switched on. Esther began pulling one string at a time in the order *ABCDEFGGFEDCBA* and thereafter repeating this order. After Esther pulled the 2011^{th} string, which light bulbs were switched on?

11. Find the value of

$$\frac{1\times2\times3 + 3\times6\times9 + 2\times4\times7 + 7\times14\times21 + 3\times4\times67}{1\times3\times5 + 3\times9\times15 + 4\times5\times7 + 7\times21\times35 + 2\times5\times201}$$

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12. Ali and Brian left Town A at the same time for Town B, which is 22.5 km away from Town A. Ali set out riding a bicycle at 8 km/h, parked the bicycle at a cycle station and walked at 5 km/h for the rest of the journey. Brian set out walking at 4 km/h, reached the cycle station and used the bicycle that was used by Ali and rode at 10 km/h. Both Ali and Brian reached Town B at the same time. For how many minutes was the bicycle not in motion?

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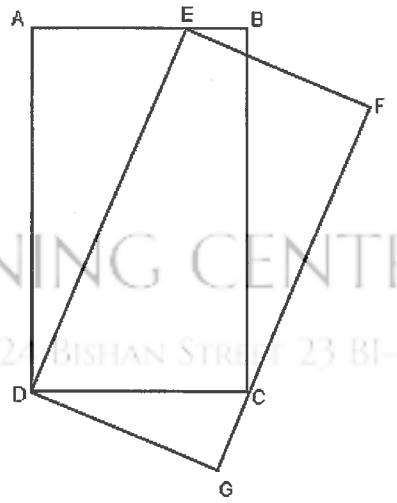
13. Find the sum

$$\frac{1}{1024}+\frac{1}{512}+\frac{1}{256}+\frac{1}{128}+\ldots+\frac{1}{2}+1+2+4+8+\ldots+1024.$$

14. In the following figure, ABCD is a rectangle with AB = 7 cm, BC = 12 cm.
DEFG is a rectangle with DE = 13 cm.
Find the length EF in cm, correct to 1 decimal place.



15. Three little sheep are erecting a fence to keep the nasty wolves away. The first two sheep, working together, could erect the fence in 2 hours. The first and third sheep, working together, could erect the fence in 1 hour and 12 minutes. The second and third sheep, working together, could complete the job in one hour and 30 minutes. How long will it take for the second sheep alone to complete the job?

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16. What is the remainder when $2^{2009} + 2009^2$ is divided by 13?

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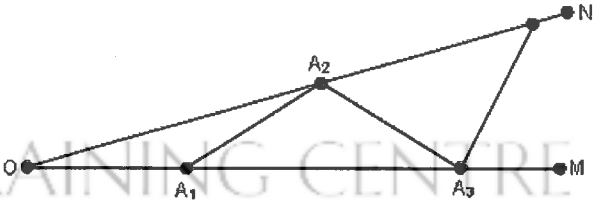
17.

The diagram below shows $\angle MON = 16°$. A_1 is a given point on OM, A_2 is chosen on ON such $A_1A_2 = OA_1$. Point A_3 is then chosen on OM such that $A_2A_3 = A_1A_2$.
This process of choosing a new point is repeated subject to the conditions that
$OA_1 < OA_3 < OA_5 <...$ and $OA_2 < OA_4 < OA_6 <....$ What is the last point at the end of this process?



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18.

Starting with the 2, the number 2009 can be formed by moving either horizontally, vertically, or diagonally from square to square in the grid. How many different paths can be followed to form 2009?

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9	9	9	9	9
9	0	0	0	9
9	0	2	0	9
9	0	0	0	9
9	9	9	9	9

19. There are 8 points marked on the circumference of a circle. If each marked point is joined to all the other marked points to form line segments, what is the largest number of points of intersection of all these line segments inside the circle?

20. Let A = (1 × 2 × 3 × ... × 2008 × 2009) + (2010 + 2011 + ... + 2028 + 2029).
Find the sum of the last 499 digits of A.