Mathlete Training Centre
Round 1 RIPMWC open

2008 RIPMWC open round 1

1.

Four teams took part in a soccer tournament. Each team played with every other team exactly once. In any single match, the winning team gets 3 points, the losing team gets no points, and in the event of a draw, both teams get 1 point each. After all the matches were played, the scores of all the teams were added up. What is the smallest possible value of this sum?

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2.

Calculate $\left(24+\frac{8}{9}\right)\times 10.375+\left(35+\frac{1}{9}\right)\times\frac{3}{8}$

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3.

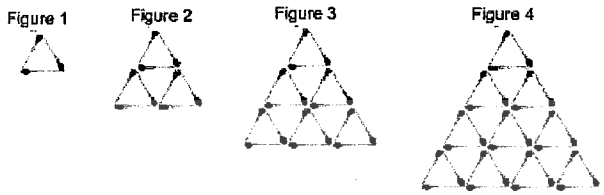
It is Wednesday today. What day would it be 20082008 days later?

4.

Given that $x = 1111111111 \times 9999999999$, find the sum of digits of x.

5.

Matchsticks are used to make a series of triangular patterns as shown below:



In figure 4, there are 16 small equilateral triangles.
Which figure will have 14400 small equilateral triangles ?

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6.

When $\frac{2008}{7}$ is expressed as a decimal, the 2008^{th} digit after the decimal point is

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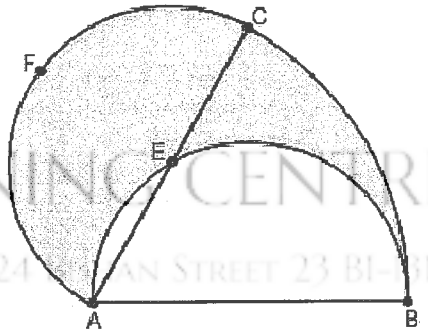
7.

20 identical apples are to be distributed to 3 girls A, B and C such that each girl gets at least 1 apple. How many ways can this be done?

8.

The diagram shows a semi-circle AEB with AB as diameter, an arc BC with centre A, radius AB and $\angle BAC = 60°$ and a semi-circle ACF with AC as diameter. Given that AB = 3 cm and taking π to be 3.14 , calculate the area of the shaded region in cm^2.



9. A monkey has a number of apples.

On the 1st day, it eats $\frac{1}{7}$ of the total number of apples.

On the 2nd day, it eats $\frac{1}{6}$ of the remaining number of apples.

On the 3rd day, it eats $\frac{1}{5}$ of the remaining number of apples.

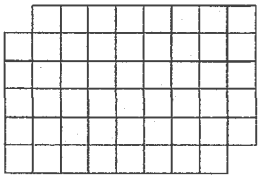
On the 4th day, it eats $\frac{1}{4}$ of the remaining number of apples.

On the 5th day, it eats $\frac{1}{3}$ of the remaining number of apples.

On the 6th day, it eats $\frac{1}{2}$ of the remaining number of apples.

If the monkey has left with 12 apples, how many apples did he eat on the 1st and 2nd days altogether?

10. A grid of squares measuring 9 units by 6 units has two corners removed as shown. How many squares of any size are contained within the grid.



11.

Five workers A, B, C, D and E work in an office for 5 days from Monday to Friday. Given that

- For each day from Monday to Friday, exactly 3 workers are at work while 2 workers are enjoying their day-off.
- No worker can be off for 2 consecutive days, neither can they work for 3 consecutive days.
- B works 2 days more than E; there is only one day when A and C work together; C works on Monday.

Who are the 3 workers who work on Friday?

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12.

What is the remainder when $1^2 + 2^2 + 3^2 + \ldots + 2007^2 + 2008^2$ is divided by 7 ?

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13.

120 boys are standing in front of their lockers, numbered from 1 to 120. At the start, all lockers are opened. A teacher calls out '1', and since 1 is a divisor of all the numbers, all the boys close their lockers. The teacher then calls out '2', and only those boys whose locker numbers are divisible by 2 will open their lockers. The teacher then calls out '3'. Those boys whose locker numbers are divisible by 3 will open their lockers if they are closed, or close their lockers if they are opened. This process is continued until '120' is called out. How many lockers will be closed at the end of the entire process?

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14.

Albert and Brian are training for a race by running up a 700 m slope. They each run up the slope at different constant speeds. Each of them doubles his uphill speed when they run down the slope. Brian reaches the top first, and immediately starts running down, meeting Albert 70 m from the top. When Brian reaches the bottom, how far behind is Albert?

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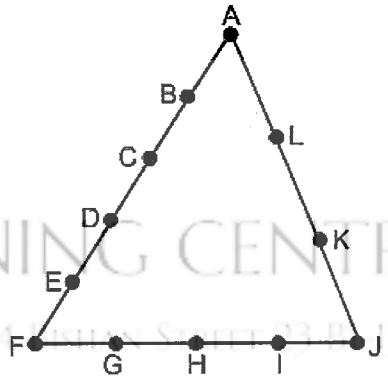
15.

By using 3 of the points from the points A, B, C, . . . , K, L as shown in the diagram to form a triangle, determine how many such triangles can be formed?



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16.

The clock on the wall shows a time which is close to 10 o'clock. Interestingly, it is also noted that the number "10" is exactly midway between the minute hand and the hour hand. The number of minutes past 9 o'clock shown by the clock is

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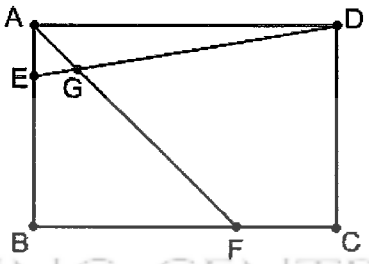
17.

The given diagram shows a rectangle ABCD whose area is 120 cm^2. Given that BE = 3AE and BF = 2FC, find the area of EGFB in cm^2.



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18.

A bag contains x balls. Each ball is either red or white. The ratio of number of red balls to white balls is 19 : 13. When an additional m red balls are added to the bag, the above ratio becomes 5 : 3. Next, without removing the m additional red balls, another additional n white balls are added to the bag. The ratio then becomes 13 : 11. Given that n – m = 80, find x.

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19. A group of 3 natural numbers which may or may not be different are such that their highest common factor is 10 and their lowest common multiple is 100. How many such groups are possible?

20. To complete a given project, workers A, B and C, each working alone, need 36 days, 30 days and 48 days respectively. At the start of the project, A, B and C worked together. After working for some days, C took y days off from work while A and B continued. As a result, A and B needed some additional days to complete the project. If the entire project took an exact number of days and y is a whole number, find y.